LAST NAME:	FIRST NAME:	CIRCLE: Akbar 4pm Coskunuzer 8:30am Coskunuzer 10am Zweck 1pm

MATH 2415 [Spring 2023] Exam II

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points. **Your points for each problem will be recorded on the top of the second page.**

(1) [9 pts] Find an equation for the tangent plane to the surface $z = 2x^2 + 4y^2$ at the point $(1, 1, 6)$.

1 /9	2 /12	3 /15	4 /15	5 /12	6 /12	T /75

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(2) [12 pts]

(a) Let $(x, y) = \mathbf{r}(t) = (\sqrt{2}\cos t, 2\sqrt{2}\sin t)$ and let $z = f(x, y)$ be a function for which $f(1, 2) = 3$, $\frac{\partial f}{\partial x}(1, 2) = 4$ and $\frac{\partial f}{\partial y}(1, 2) = 5$. Let $g(t) = f(\mathbf{r}(t))$. Find $g'(\frac{\pi}{4})$.

(b) Show that the function $u(x, y) = x^3 - 3xy^2$ satisfies Laplace's equation, $u_{xx} + u_{yy} = 0$.

(3) [15 pts] Let $f(x, y) = xe^y$.

(a) Find the directional derivative of $f(x, y) = xe^y$ at the point $(2, 0)$ in the direction of the vector $3\mathbf{i} + 4\mathbf{j}$.

(b) In what direction does f have the maximum rate of change at the point $(2, 0)$? What is this maximum rate of change?

(c) In what directions is the rate of change of f at the point equal to zero at the point $(2, 0)$?

(4) [15 pts] Let S be the surface with parametrization

$$(x, y, z) \; = \; \mathbf{r}(u, v) \; = \; \left(\sqrt{1+v^2}\cos u\,,\; \sqrt{1+v^2}\sin u\,,\; \frac{v}{2}\right), \qquad \text{for } 0 \le u \le 2\pi \text{ and } -2 \le v \le 2.$$

(a) Show that S is part of a hyperboloid of one sheet. **Hint:** Find an equation of the form $F(x, y, z) = 0$ for this surface.

(b) Sketch the surface S, together with the grid curves where (i) $u = \frac{\pi}{2}$ and (ii) $v = 0$. (Label these curves!)

(c) Calculate a tangent vector to the grid curve where $u = \frac{\pi}{2}$ at the point $\mathbf{r}(\frac{\pi}{2}, 0)$.

(5) [12 pts] Find and classify all critical points of the function $f(x, y) = 2x^2 + y^4 + 4xy$.

(6) [12 pts] Use the method of Lagrange multipliers to find the absolute maximum and minimum of the function $f(x, y) = xy$ on the ellipse $x^2 + 4y^2 = 8$.