LAST NAME:	FIRST NAME:	CIRCLE: Akbar 4pm, Coskunuzer 8:30am, Coskunuzer 10am, Zweck 1pm

MATH 2415 [Spring 2023] Exam I

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points. **Your points for each problem will be recorded on the top of the second page.**

(1) [12 pts] Let $\mathbf{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$, and $\mathbf{w} = 5\mathbf{i} + 9\mathbf{j} - 2\mathbf{k}$.

(a) Find the vector projection of $\mathbf{v}$ onto $\mathbf{u}$.

(b) Find the volume of the parallelepiped determined by the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

1 /12	2 /12	3 /13	4 /12	5 /14	6 /12	T /75

(2) [12 pts] Consider the three points $P = (1, 0, 1)$, $Q = (-2, 1, 3)$, and $R = (4, 2, 5)$.
(a) Find a unit vector that is perpendicular to the plane containing these three points.

(b) Find the area of the triangle PQR.

(3) [13 pts]
(a) Suppose that two lines, L_1 and L_2, intersect in a point. Draw a schematic diagram that illustrates why there is exactly one plane $\mathcal{P}$ than contains both L_1 and L_2. Write a brief explanation of your diagram.

For the rest of the problem suppose that L_1 is the line parametrized by $\mathbf{r}_1(t) = (2t+1, 3t-2, t+1)$ and L_2 is the line parametrized by $\mathbf{r}_2(s) = (s+1, 2s-3, 3s-4)$.

(b) Show that the point $\mathbf{p} = (3, 1, 2)$ lies on both lines.

(c) Find an equation of the form $Ax + By + Cz = D$ for the plane $\mathcal{P}$ than contains both L_1 and L_2.

(4) [12 pts]
(a) Sketch the surface whose equation in spherical coordinates, (ρ, θ, ϕ), is $\phi = \pi/4$.

(b) Convert the equation $\phi = \pi/4$ to rectangular coordinates.

(c) Convert the equation $\phi = \pi/4$ to cylindrical coordinates.

(5) [14 pts]
(a) Sketch the surface $x^2 + z^2 - 2y^2 = 4$. **Hint:** You may find it helpful to use cylindrical coordinates (r, θ, y) (instead of the usual (r, θ, z)).

(b) Make *labelled* sketches of the traces (slices) of the surface $x = z^2 - 4y^2$ in the planes $z = 0$, $y = 0$, and $x = k$, for $k = 0, 1, -2$. (You do not need to sketch the surface itself.)

(6) [12 pts] Let C be the curve parametrized by $\mathbf{r}(t) = (3\sin t, 4, 3\cos t)$.
(a) Show that C lies on a cylinder and on a sphere.

(b) Find a parametrization for the tangent line to C at the point where $t = \pi/3$.