LAST NAME:	FIRST NAME:	CIRCLE:	Coskunuzer 8:30am	Ahsan 1pm
			Ahsan 2:30pm	Zweck 4pm

MATH 2415 [Fall 2024] Exam II

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. This 75 minute exam is worth 75 points. **Points will be recorded on the top of the second page.**

(1) [12 pts] Suppose that $z = f(x,y) = \cos(2x + 3y)$ where $x = x(t)$ and $y = y(t)$. If $x(0) = -\pi/4$, $y(0) = \pi/3$, $x'(0) = 5$, and $y'(0) = 4$, find $\frac{dz}{dt}$ at $t = 0$.

1 /12	2 /13	3 /13	4 /12	5 /13	6 /12	T /75

(2) [13 pts] Let $f(x,y) = x^2 + \cos y + 2ye^x$ and let $\mathbf{x}_0 = (0, \frac{\pi}{2})$.

(a) Find the gradient of f at $\mathbf{x}_0$.

(b) Find the directional derivative of f at $\mathbf{x}_0$ in the direction of the vector $\mathbf{v} = (4,3)$.

(c) Find the minimum (i.e. most negative) rate of change of f at $\mathbf{x}_0$ and the direction in which it occurs.

(d) Which of the following vectors (if any) are tangent to the curve $f(x,y) = \pi$ at the point $\mathbf{x}_0$: $\mathbf{u} = (1,\pi)$, $\mathbf{v} = (1,-\pi)$, and $\mathbf{w} = (2\pi, 2)$. Explain!

(3) [13 pts] Find and classify all critical points of the function $f(x, y) = y^3 - 12y + 3x^2y + 3x^2$.

(4) [12 pts]
(a) Suppose $f_{xx}(0,0) = -1$, $f_{yy}(0,0) = 2$ and $f_{xy}(0,0) = 3$. Which would you expect to be larger (more positive) and why:

(i) $f_y(0,0)$ or $f_y(0,0.1)$?

(ii) $f_x(0,0)$ or $f_x(0,0.1)$?

(b) Is there a function $z = f(x,y)$ so that $\frac{\partial f}{\partial x} = 2x\cos y$ and $\frac{\partial f}{\partial y} = x^2 \sin y$? Explain!

(5) [13 pts] Consider the parametrized surface $(x, y, z) = \mathbf{r}(u, v) = (\sqrt{2}\cos u, v, \sqrt{2}\sin u)$.

(a) Find an equation of the form $F(x, y, z) = 0$ for this surface.

(b) Find an equation of the tanget plane to the surface at the point $\mathbf{r}(\frac{\pi}{4}, 1)$. Write your answer in the form $ax + by + cz + d = 0$.

(6) [12 pts] Use the Method of Lagrange Multipliers to find the absolute maximum and absolute minimum of the function $f(x, y) = 2x^2 + 3y^2$ subject to the constraint $x^2 + y^2 = 1$.