LAST NAME:	FIRST NAME:	CIRCLE:	Coskunuzer 8:30am	Ahsan 1pm
			Ahsan 2:30pm	Zweck 4pm

MATH 2415 [Fall 2024] Exam I

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. This 75 minute exam is worth 75 points. **Points will be recorded on the top of the second page.**

(1) [12 pts] Let $\mathbf{u} = \langle 2, 1, -3 \rangle$ and $\mathbf{v} = \langle -1, 2, 1 \rangle$.
(a) Find the scalar projection of $\mathbf{u}$ onto $\mathbf{v}$.

(b) Find the vector projection of $\mathbf{v}$ onto $\mathbf{u}$.

(c) Find a unit vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

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(2) [13 pts] Consider the points $A = (3, -1, 1)$, $B = (4, 2, -1)$ and $C = (2, 3, -3)$. Let AB and AC are two adjacent sides of a parallelogram $ABCD$.

(a) Find the coordinates of the point D.

(b) Find the area of the parallelogram $ABCD$.

(c) Consider the point $E = (4, -5, 5)$. Show that the vector $\overrightarrow{AE}$ lies in the same plane as the parallelogram $ABCD$.

(3) [13 pts] Let $\mathcal{P}$ be the plane through the point $A = (1, 0, 2)$ that is perpendicular to the line with parameterization $\mathbf{r}(t) = \mathbf{q} + t\mathbf{v} = (-1 + 3t)\mathbf{i} + (4 - 2t)\mathbf{j} + 4t\mathbf{k}$.

(a) Draw a schematic diagram showing the relationship between the plane and the line. Include the point, A, and the vectors, $\mathbf{q}$ and $\mathbf{v}$ in your sketch.

(b) Find an equation of the form $Ax + By + Cz = D$ for the plane, $\mathcal{P}$.

(c) Find a parameterization of the plane, $\mathcal{P}$.

(4) [12 pts]
(a) Let P be the point with rectangular coordinates $(x, y, z) = (-1, 1, \sqrt{2})$.

(i) Find the cylindrical coordinates of P.

(ii) Find the spherical coordinates of P.

(b) Convert the equation $\phi = \pi/4$ in spherical coordinates (ρ, θ, ϕ), into an equation involving cylindrical coordinates (r, θ, z).

(5) [13 pts] Let $(x, y, z) = \mathbf{r}(t) = (t^2 + 3t, e^{2t}, \sin t)$ be the position of a particle at time t.

(a) Find the velocity vector of the particle at time t.

(b) Find the speed of the particle at time $t = 0$.

(c) Find a parametrization for the tangent line to the particle's motion at the point where $t = 0$.

(6) [12 pts] Make labelled sketches of the traces (slices) of the surface $z^2 = 1 + y^2 + 4x^2$ in the planes $x = 0$, $y = 0$, and $z = k$ for $k = 0, \pm 1, \pm 2$. Be sure to include any asymptotes and intercepts in your sketches. Then make a labelled sketch of the surface.