LAST NAME:	FIRST NAME:	CIRCLE:	Khoury 5:30pm	Coskunuzer 8:30am
		Coskunuzer 11:30am	Zweck 1pm	Zweck 4pm

1 /10	2 /10	3 /10	4 /10	5 /10	
6 /10	7 /10	8 /10	9 /10	10 /10	T /100

MATH 2415 Final Exam, Fall 2023

No books or notes! **NO CALCULATORS! Show all work and give complete explanations**. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts] Find an equation of the form $Ax + By + Cz = D$ for the plane containing the points $(0, 1, 1)$, $(1, 0, 1)$, and $(1, 1, 0)$. Which of the following points lies on this plane: $P = (2, -1, 3)$, $Q = (3, -4, 3)$.

(2) [10 pts]
(a) Find the volume of the parallelipiped determined the the vectors $\mathbf{u} = (1, 4, 2)$, $\mathbf{v} = (-1, 1, 4)$ and $\mathbf{w} = (5, 1, 2)$.

(b) Make a sketch that shows how to project the vector $\mathbf{v} = \mathbf{i}$ onto the vector $\mathbf{w} = \mathbf{i} + \sqrt{3}\mathbf{j}$. Use your sketch to find the component of $\mathbf{v}$ in the direction $\mathbf{w}$.

(3) [10 pts]
(a) Sketch the surface $x^2 + y^2 - z^2 = 2$ for $-\sqrt{2} \leq z \leq \sqrt{2}$.

(b) Show that the line through the point $(1, 1, 0)$ in the direction of the vector $(-1, 1, \sqrt{2})$ lies on the surface in (a).

(4) [10 pts] Find and classify all critical points of the function

$$f(x, y) = 3x^2y - 3y + y^3$$

(5) [10 pts] Evaluate the double integral $\int_{x=0}^{x=1} \int_{y=\sqrt{x}}^{y=1} \sqrt{y^3+1}\, dydx$.

(6) [10 pts] Let E be the solid in the first octant that is bounded by the planes $x + z = 2$ and $2y + z = 2$. Calculate $\iiint_E z\, dV$. *Hint:* It may be helpful to sketch E.

(7) [10 pts]
(a) Find $\iiint_E \sqrt{x^2+y^2}\, dV$ where E is the solid above the xy-plane, below the paraboloid $z = 9 - x^2 - y^2$ and *outside* the cylinder $x^2 + y^2 = 4$.

(b) Use a triple integral in spherical coordinates to find the volume of the sphere of radius R.

(8) [10 pts] Use the change of variables theorem to evaluate $\iint_R (x+y)^2 \mathrm{e}^{x-y}\, dxdy$ where R is the parallelogram bounded by $x+y=2$, $x+y=5$, $x-y=-2$ and $x-y=1$.

(9) [10 pts]
(a) Let C be the semicircle $x^2 + y^2 = 9$ with $y \geq 0$ oriented counter-clockwise and let $\mathbf{F}$ be the vector field $\mathbf{F} = -y\mathbf{i} + x\mathbf{j}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(b) Let C be the curve $y = \sqrt[3]{x+1}$ from $(-1, 0)$ to $(0, 1)$. Evaluate

$$\int_C (2xy + 3)dx + (x^2 + 10y)dy$$

(10) [10 pts] Let D be the triangular domain with vertices $(0,0), (0,4)$ and $(2,4)$. Let C be the boundary of D, oriented counter clockwise. Evaluate

$$\int_C (\sqrt{x^3-1} + 2xy^2)dx + (x^2y - e^y)dy.$$