LAST NAME:	FIRST NAME:	CIRCLE:	Khoury 5:30pm	Coskunuzer 8:30am
		Coskunuzer 11:30am	Zweck 1pm	Zweck 4pm

MATH 2415 [Fall 2023] Exam II

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. This 75 minute exam is worth 75 points. **Points will be recorded on the top of the second page.**

(1) [10 pts] Suppose that $z = f(x, y) = \cos(3x + 2y)$ where $x = x(t)$ and $y = y(t)$. If $x(0) = \pi/3$, $y(0) = -\pi/4$, $x'(0) = 4$, and $y'(0) = 5$, find $\frac{dz}{dt}$ at $t = 0$.

1	/10	2	/14	3	/13	4	/12	5	/13	6	/13	T	/75

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(2) [14 pts]
Let $f(x, y) = x^2y^2 - 2x - 2y$ and let $\mathbf{x}_0 = (2, 1)$.

(a) Find the gradient of f at $\mathbf{x}_0$.

(b) Find the directional derivative of f at $\mathbf{x}_0$ in the direction of the vector $\mathbf{v} = (4, 3)$.

(c) Find the maximum rate of change of f at $\mathbf{x}_0$ and the direction in which it occurs.

(d) Let C be the level curve $f(x, y) = -2$. Find a tangent vector to the curve C at the point $\mathbf{x}_0$.

(3) [13 pts] (a) Let $z = f(x, y) = x^2 + 4y^2$. By calculating an equation for the tangent plane to the graph of f at an appropriate point, find an approximation to $f(1.99, 0.99)$.

(b) Suppose that $z = f(x, y)$ is a function such that $\frac{\partial f}{\partial x}(4, y) = y^2 + 3y + 1$. Let $g(x) = \frac{\partial f}{\partial y}(x, 5)$. What is the rate of change of g at $x = 4$? [Do not attempt to find a formula for f].

(4) [12 pts] Consider the parameterized surface

$$(x, y, z) \;=\; \mathbf{r}(\theta, \phi) \;=\; (3\cos\theta\,\sin\phi, 3\sin\theta\,\sin\phi, 3\cos\phi) \qquad \text{for } 0 \le \theta \le 2\pi, \quad 0 \le \phi \le \pi/2.$$

(a) Find an equation of the form $F(x, y, z) = 0$ for this surface.

(b) Sketch the surface and the curve where $\phi = \pi/4$.

(5) [13 pts] Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - xy + y$ on the rectangle $[0, 2] \times [0, 3]$.

(6) [13 pts] Use the Method of Lagrange Multipliers to find the absolute maximum and minimum values of the function $f(x, y) = x^2 y$ subject to the constraint $x^2 + y^2 = 3$.