LAST NAME:	FIRST NAME:	CIRCLE:	Khoury 5:30pm	Coskunuzer 8:30am
		Coskunuzer 11:30am	Zweck 1pm	Zweck 4pm

MATH 2415 [Fall 2023] Exam I

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. This 75 minute exam is worth 75 points. **Points will be recorded on the top of the second page.**

(1) [13 pts] Let $A = (1, 2, -3)$, $B = (4, 8, 0)$ and $C = (7, -1, 6)$.

(a) Find the point D on the line segment from A to B for which $2|\overrightarrow{AD}| = |\overrightarrow{DB}|$. **Hint:** Parametrize the line segment from A to B.

(b) Find the area of the triangle ABC.

1 /13	2 /12	3 /13	4 /12	5 /13	6 /12	T /75

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(2) [12 pts] Let $\mathbf{u} = \langle 3, -2, 1 \rangle$ and $\mathbf{v} = \langle 5, 4, 3 \rangle$.
(a) Find the scalar projection of $\mathbf{v}$ onto $\mathbf{u}$.

(b) Find the vector projection of $\mathbf{u}$ onto $\mathbf{v}$.

(c) Let $\mathbf{w} = \langle 1, 7, a \rangle$. If the vector projection of $\mathbf{w}$ onto $\mathbf{v}$ is $\mathbf{0}$ (the zero vector), find a.

(3) [13 pts] Let $\mathcal{P}$ be the plane through the point $A = (2, 1, 0)$ that is perpendicular to the line with parameterization $\mathbf{r}(t) = \mathbf{q} + t\mathbf{v} = (2 + 5t)\mathbf{i} + (2 - t)\mathbf{j} + 4t\mathbf{k}$.

(a) Draw a schematic diagram showing the relationship between the plane and the line. Include the point, A, and the vectors, $\mathbf{q}$ and $\mathbf{v}$ in your sketch.

(b) Find an equation of the form $Ax + By + Cz = D$ for the plane, $\mathcal{P}$.

(c) Find a parameterization of the plane, $\mathcal{P}$.

(4) [12 pts]
(a) Let P be the point with spherical coordinates $(\rho, \theta, \phi) = (2, \pi/3, \pi/4)$.

(i) Find the cylindrical coordinates of P.

(ii) Find the rectangular coordinates of P.

(b) Convert the equation $z = \sqrt{3x^2 + 3y^2}$ into an equation involving spherical coordinates ρ, θ and ϕ.

(5) [13 pts] Let C be the parametrized curve given by $(x, y) = \mathbf{r}(t) = (\sin t, 2\cos t)$ for $0 \leq t \leq \pi/2$.
(a) Eliminate t to obtain an equation relating x and y.

(b) Sketch the curve, clearly marking the start and end points and the direction of motion.

(c) Find a parametrization for the tangent line to the curve C at the point where $t = \pi/4$.

(d) Find constants a and b and a function $f(t)$ so that the length of C is $L = \int_a^b f(t)\,dt$, where C is the curve whose parametrization, $(x, y) = \mathbf{r}(t)$, is given above.

(6) [12 pts] Make labelled sketches of the traces (slices) of the surface $x^2 - y^2 + 4z^2 = 1$ in the planes $x = 0$, $z = 0$, and $y = k$ for $k = 0, \pm 1, \pm 2$. Be sure to include any asymptotes and intercepts in your sketches. Then make a labelled sketch of the surface.