LAST NAME:	FIRST NAME:	CIRCLE:	Coskunuzer 8:30am	Dahal 11:30am
		Dahal 5:30pm	Zweck 1pm	Zweck 4pm

MATH 2415 [Fall 2022] Exam II, Nov 4th

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points. **Your points for each problem will be recorded on the top of the second page.**

(1) [12 pts]

(a) Suppose that $w = f(x, y, z)$, where $x = x(t)$, $y = y(t)$ and $z = z(t)$. Use a tree diagram to write out a formula for $\frac{dw}{dt}$. Use this formula to find $\frac{dw}{dt}$ at $t = 0$ when $f(x, y, z) = e^{x^2+y^2+z}$, $x = t^3$, $y = \sin t$, $z = 3t$.

(b) Find the equation for the tangent plane to the graph of $z = f(x, y) = y^2e^x$ at $(0, 1)$. Use this equation to approximate $f(0.2, 1.1)$.

1	/12	2	/10	3	/14	4	/15	5	/12	6	/12	T	/75

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(2) [10 pts]
(a) Show that $u(x, y) = \ln(x^2 + y^2)$ satisfies the partial differential equation $u_{xx} + u_{yy} = 0$.

(b) Let $f(x, y) = xy$. As x_0 increases from $x_0 = 1$, does the slope of the slice of the graph of f in the plane $x = x_0$ increase or decrease? Why?

(3) [14 pts] Let $f(x, y) = ye^{2x}$.
(a) Find the gradient of f at the point $(0, 1)$.

(b) Find the directional derivative of f at the point $(0, 1)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + \mathbf{j}$.

(c) Find the direction of the minimum rate of change in f at (0,1). Also find the minimum rate of change.

(d) Sketch the level curve $f(x, y) = 1$. Add the vector $\nabla f(0, 1)$ to your sketch.

(4) [15pts] Find and classify all critical points of the function $f(x, y) = 2x^3 - 3x^2y + 3y^2 + 12x^2$.

(5) [12 pts] Let S be the surface with parametrization

$$(x, y, z) \;=\; \mathbf{r}(u, v) \;=\; (u\cos v, u\sin v, u^2), \qquad \text{for } 0 \le u \le 3 \text{ and } 0 \le v \le 2\pi.$$

(a) Show that S is part of a paraboloid. **Hint:** Find an equation of the form $F(x, y, z) = 0$ for this surface.

(b) Sketch the surface S, together with the grid curves where (i) $u = 2$ and (ii) $v = \frac{\pi}{4}$. (Label these curves!)

(c) Calculate a tangent vector to the grid curve where $v = \frac{\pi}{4}$ at the point $\mathbf{r}(2, \frac{\pi}{4})$.

(6) [12 pts] Use the method of Lagrange multipliers to find the absolute maximum and minimum of the function $f(x, y) = (x + 3)^2 + (y - 3)^2$ on the circle $x^2 + y^2 = 8$.