LAST NAME:	FIRST NAME:	CIRCLE:	Coskunuzer 8:30am	Dahal 11:30am
		Dahal 5:30pm	Zweck 1pm	Zweck 4pm

MATH 2415 [Fall 2022] Exam II, Nov 4th

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points. **Your points for each problem will be recorded on the top of the second page.**

- (1) [12 pts]
- (a) Suppose that w = f(x, y, z), where x = x(t), y = y(t) and z = z(t). Use a tree diagram to write out a formula for $\frac{dw}{dt}$. Use this formula to find $\frac{dw}{dt}$ at t = 0 when $f(x, y, z) = e^{x^2 + y^2 + z}$, $x = t^3$, $y = \sin t$, z = 3t.

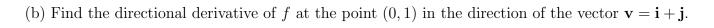
⁽b) Find the equation for the tangent plane to the graph of $z = f(x, y) = y^2 e^x$ at (0, 1). Use this equation to approximate f(0.2, 1.1).

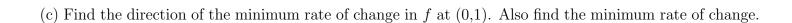
1	/12	2 /	/10	3 /14	4 /15	1.5 /11	$i \mid 0 \qquad /12$	T /75

- (2) [10 pts]
- (a) Show that $u(x,y) = \ln(x^2 + y^2)$ satisfies the partial differential equation $u_{xx} + u_{yy} = 0$.

(b) Let f(x,y) = xy. As x_0 increases from $x_0 = 1$, does the slope of the slice of the graph of f in the plane $x = x_0$ increase or decrease? Why?

 (3) [14 pts] Let f(x,y) = ye^{2x}. (a) Find the gradient of f at the point (0,1). 								





(d) Sketch the level curve f(x,y)=1. Add the vector $\nabla f(0,1)$ to your sketch.

(4) [15pts] Find and classify all critical points of the function $f(x,y)=2x^3-3x^2y+3y^2+12x^2$.

(5) [12 pts] Let S be the surface with parametrization

$$(x, y, z) = \mathbf{r}(u, v) = (u \cos v, u \sin v, u^2),$$
 for $0 \le u \le 3$ and $0 \le v \le 2\pi$.

(a) Show that S is part of a paraboloid. Hint: Find an equation of the form F(x, y, z) = 0 for this surface.

(b) Sketch the surface S, together with the grid curves where (i) u=2 and (ii) $v=\frac{\pi}{4}$. (Label these curves!)

(c) Calculate a tangent vector to the grid curve where $v = \frac{\pi}{4}$ at the point $\mathbf{r}(2, \frac{\pi}{4})$.

(6) [12 pts] Use the method of Lagrange multipliers to find the absolute maximum and minimum of the function $f(x,y) = (x+3)^2 + (y-3)^2$ on the circle $x^2 + y^2 = 8$.