LAST NAME:	FIRST NAME:	CIRCLE:	Coskunuzer 8:30am	Dahal 11:30am
		Dahal 5:30pm	Zweck 1pm	Zweck 4pm

1 /12	2 /12	3 /12	4 /15	5 /12	6 /12	T /75

MATH 2415 [Fall 2022] Exam I, Sep 30th

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points.

(1) [12 pts] Let $\mathbf{u} = 4\mathbf{i} + 3\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

(a) Find the scalar projection of $\mathbf{u}$ onto $\mathbf{v}$.

(b) Find the vector projection of $\mathbf{v}$ onto $\mathbf{u}$.

(c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$. [Your answer should be in terms of an inverse trigonometric function.]

(2) [12 pts] Let $\mathbf{u} = (3, 0, -2)$ and $\mathbf{v} = (-4, 1, 2)$.
(a) Find a vector $\mathbf{w}$ that has length one and is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

(b) Find the volume of the parallelepiped generated by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{k} = (0, 0, 1)$.

(3) [12 pts] Let C be the curve parametrized by $\mathbf{r}(t) = \langle \cos t, \sin t, \ln(\cos t) \rangle$.
(a) Find a parametrization of the line tangent to the curve, C, when $t = \frac{\pi}{4}$.

(b) Show that the length of the segment of the curve, C, from $t = 0$ to $t = \frac{\pi}{4}$ is $L = \int_0^{\pi/4} \sec t \, dt$.

(4) [15pts]
(a) Parametrize the curve of intersection of the surfaces $x = y^2 - z^2$ and $y^2 + z^2 = 9$.

(b) Let P be the point with spherical coordinates $(\rho, \theta, \phi) = \left(4, -\frac{\pi}{4}, \frac{\pi}{3}\right)$. Find the rectangular coordinates of P.

(c) Identify and sketch the surface which is given in cylindrical coordinates by the equation $z^2 - r^2 = 4$.

(5) [12 pts] (a) Let P be the plane parametrized by $\mathbf{r}(s,t) = (1+2s-4t, 3s+t, 6-t)$. Find an equation of the form $Ax+By+Cz=D$ for the plane, P.

(b) Consider the lines, L_1, L_2, and L_3 parametrized by

$L_1 : \mathbf{r}_1(t) = (2+5t, -1+4t, t)$, $\quad L_2 : \mathbf{r}_2(t) = (2+3t, 3+4t, 1-t)$, $\quad L_3 : \mathbf{r}_3(t) = (5+3t, 2-4t, 3+t)$.

Let $\mathcal{P}$ be a plane that is perpendicular to L_1. Could $\mathcal{P}$ contain the line L_2? Could $\mathcal{P}$ contain the line L_3?

(6) [12 pts] Make a labelled sketch of the traces (slices) of the surface

$$x^2 - y^2 + 4z^2 = 0$$

in the planes $x = 0$, $z = 0$, and $y = k$ for $k = 0, \pm 1, \pm 2$. Then make a labelled sketch of the surface.