LAST NAME:	FIRST NAME:	CIRCLE: Eydelzon Coskunuzer Dahal Zweck 1pm Zweck 4pm

1 /10	2 /10	3 /10	4 /10	5 /10
6 /10	7 /10	8 /10	9 /10	10 /10

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MATH 2415 Final Exam, Fall 2021

No books or notes! **NO CALCULATORS! Show all work and give complete explanations**. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts]
(a) Find an equation of the form $Ax + By + Cz = D$ for the plane that goes through the point $(5, -1, 0)$ and is perpendicular to the line with parameterization $(x, y, z) = \mathbf{r}(t) = (1 + 2t, 3 - 7t, -2 + 3t)$.

(b) Let $\mathbf{u}$ and $\mathbf{v}$ be two unit vectors. What are the possible angles between $\mathbf{u}$ and $\mathbf{v}$ if $|\mathbf{u} \times \mathbf{v}| = \frac{1}{2}$?

(2) [10 pts] Let $\mathbf{u} = \langle 4, 2, -2 \rangle$, $\mathbf{v} = \langle 1, -1, 3 \rangle$ and $\mathbf{w} = \langle 1, -1, 1 \rangle$.

(a) Find the vector projection of $\mathbf{v}$ onto $\mathbf{w}$.

(b) Find the area of triangle determined by the vectors $\mathbf{u}$ and $\mathbf{v}$.

(c) Determine the volume of parallelepiped determined by the vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(3) [10 pts]

(a) Evaluate the double integral

$$\int_0^8 \int_{\sqrt[3]{y}}^2 \cos(x^4)\,dx\,dy.$$

(b) Evaluate the line integral $\int_C xe^{yz}\,ds$, where C is the line segment from $(0,0,0)$ to $(1,2,3)$.

(4) [10 pts] Let S be the surface with parametrization

$$(x, y, z) \;=\; \mathbf{r}(\phi, \theta) \;=\; (\sin\phi\cos\theta, 2\sin\phi\sin\theta, 3\cos\phi), \qquad 0 \le \phi \le \pi,\ 0 \le \theta \le 2\pi.$$

(a) Show that S is an ellipsoid. Hint: Find an equation of the form $F(x, y, z) = 0$ for this surface by eliminating ϕ and θ from the equations for x, y, and z above.

(b) Calculate a normal vector to the ellipsoid at the point where $(\phi, \theta) = (\pi/4, \pi/3)$.

(5) [10 pts] Use the method of Lagrange multipliers to find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + y^2$ on the ellipse $(x + 1)^2 + 3y^2 = 4$.

(6) [10 pts] Let E be the solid in the first octant bounded by the surfaces $z = y$ and $y = 1 - x^2$. (Recall that the first octant is where $x \geq 0$, $y \geq 0$, $z \geq 0$.) Evaluate

$$\iiint_E xz\, dV.$$

(7) [10 pts] Let E be the solid bounded by the surfaces $z = 2\sqrt{x^2 + y^2}$ and $z = x^2 + y^2$. Evaluate the integral

$$\iiint_E \sqrt{x^2 + y^2}\, dV.$$

(8) [10 pts] Let R be the domain bounded by the lines $y = 0$, $y = 3$, $y = 2x - 1$, and $y = 2x - 4$. Use the change of variables $u = 2x - y$ and $v = y$ to evaluate

$$\iint_R \frac{\cos y}{(2x - y)^2}\, dA.$$

(9) [10 pts] Let $\mathbf{F}(x, y) = \sin y\,\mathbf{i} + (x\cos y - \sin y)\mathbf{j}$. Verify that the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path. Find a potential function for $\mathbf{F}$ and use it to find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is any curve from $(2, 0)$ to $(1, \pi)$.

(10) [10 pts] Let $\mathbf{F}(x, y) = x^3\mathbf{i} - y^3\mathbf{j}$ be the velocity vector field of a fluid flowing in $\mathbb{R}^2$.

(a) Calculate $\nabla \cdot \mathbf{F}$.

(b) Calculate $\nabla \times \mathbf{F}$.

(c) On average, is the fluid rotating clockwise, counter-clockwise, or not rotating at all about the point $(1, 2)$? Why?

(d) On average, is the fluid flowing in, out, or neither in or out, of a small disc centered at $(1, 2)$? Why?