LAST NAME:	FIRST NAME:	CIRCLE: Eydelzon Coskunuzer Dahal Zweck 1pm Zweck 4pm

1 /10	2 /12	3 /12	4 /8	5 /9	6 /12	7 /12	T /75

MATH 2415 [Fall 2021] Exam II, Oct 29th

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points.

(1) [10 pts]
(a) Suppose that $w = f(x, y)$, where $x = g(s, t)$ and $y = h(s, t)$. Write the chain rule formula for $\frac{\partial w}{\partial s}$.

(b) Let $w = \sin(x^2 + y^2)$, where $x = s^2 t,\ y = st^2$. Use your answer to (a) to find $\frac{\partial w}{\partial s}$ at $(s, t) = (-1, 2)$.

(2) [12 pts] Let $z = f(x, y) = \sqrt{9 + x^2 y^2}$

(a) Find an equation of the form $z = Ax + by + C$ for the tangent plane to the surface $z = f(x, y)$ at a point where $x = 2$ and $y = 2$.

(b) Use linear approximation to approximate the value of $f(2.1, 1.8)$.

(3) [12 pts] Let $f(x, y) = (x + 1)y^2e^{-x^2}$.

(a) Calculate the directional derivative of f at the point $(x, y) = (0, 1)$ in the direction of the vector $\mathbf{v} = -\mathbf{i} + \mathbf{j}$.

(b) What is the direction of steepest ascent at $(x, y) = (0, 1)$, and what is the rate of change of f in that direction?

(c) Let C be the level curve $f(x, y) = 1$. Find the slope of the tangent line to C at the point $(x, y) = (0, 1)$.

(4) [8 pts] Show that the function $f(x,t) = e^{-t}\cos\left(\frac{x}{2}\right)$ satisfies heat equation $f_t = 4f_{xx}$.

(5) [9 pts] Select the answer that is a parametrization of the double cone $x^2 + y^2 = z^2$. *Explain!!*

(I) $(x,y,z) = \mathbf{r}(u,v) = (u, \cos v, \sin v)$ for $-\infty < u < \infty$ and $0 \le v \le 2\pi$

(II) $(x,y,z) = \mathbf{r}(u,v) = (u, v, \sqrt{u^2+v^2})$ for $-\infty < u < \infty$ and $-\infty < v < \infty$

(III) $(x,y,z) = \mathbf{r}(u,v) = (u\cos v, u\sin v, u)$ for $-\infty < u < \infty$ and $0 \le v \le 2\pi$

(6) [12 pts] Find and classify all critical points of the function $f(x, y) = x^3 - 6xy + y^2$.

(7) [12 pts] Find the absolute maximum and absolute minimum of the function $f(x, y) = x^3 - 6xy + y^2$ on the rectangle $0 \leq x \leq 1$, $0 \leq y \leq 4$. [*You may use your answer to Question (6).*]