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1 /12	2 /12	3 /13	4 /12	5 /13	6 /13	T /75

MATH 2415 [Fall 2021] Exam I, Oct 1st

No books or notes! **NO CALCULATORS!** Show all work and give **complete explanations**. Don't spend too much time on any one problem. This 75 minute exam is worth 75 points.

(1) [12 pts] Let $\mathbf{u} = -\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j}$
(a) Find the vector projection of $\mathbf{u}$ onto $\mathbf{v}$.

(b) Let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$. Is $0 \leq \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} \leq \theta < \pi$? Why?

(2) [12 pts] Let $\mathcal{P}$ be the plane that contains the points $P = (1, 0, 2)$, $Q = (4, 1, -2)$, and $R = (2, 0, 0)$.
(a) Find two different vectors that are perpendicular to the plane $\mathcal{P}$ and have length 6.

(b) Find the area of the triangle whose vertices are P, Q,and R.

(3) [13 pts]
(a) Sketch the surface given in cylindrical coordinates by $r = \sqrt{2z}$.

(b) Convert the point $(-\sqrt{3}, \sqrt{3}, \sqrt{2})$ from rectangular coordinates to spherical coordinates.

(c) Find an equation in rectangular coordinates for the surface given in spherical coordinates by $\rho \sin \phi = 1$. Describe the surface.

(4) [12 pts]
(a) Let C be the curve parameterized by

$$\mathbf{r}(t) = \ln(t+1)\mathbf{i} + e^t\mathbf{j} + 2\cos t\mathbf{k}.$$

Find parametric equations for the tangent line to C at the point $P = (0, 1, 2)$.

(b) Find the length of the curve $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + t\mathbf{k}$ between the points $P = (2, 0, 0)$ and $Q = (-2, 0, \pi)$.

(5) [13 pts]
(a) Find an equation for the plane that goes through the point $(1, -3, 2)$ and is parallel to the plane whose equation is $x + 4y + 5z = 0$.

(b) Let $\mathcal{L}$ be the line of intersection of the planes $x+y+z = 3$ and $x-y+4z = 5$. Find a parameterization of the plane that contains the point $P = (1, 2, 0)$ and the line $\mathcal{L}$.

(6) [13 pts] Make a labelled sketch of the traces (slices) of the surface

$$y^2 - \frac{x^2}{3} - \frac{z^2}{12} = 1$$

in the planes $x = 0$, $z = 0$, and $y = k$ for $k = 0, \pm 1, \pm 2$. Then make a labelled sketch of the surface.