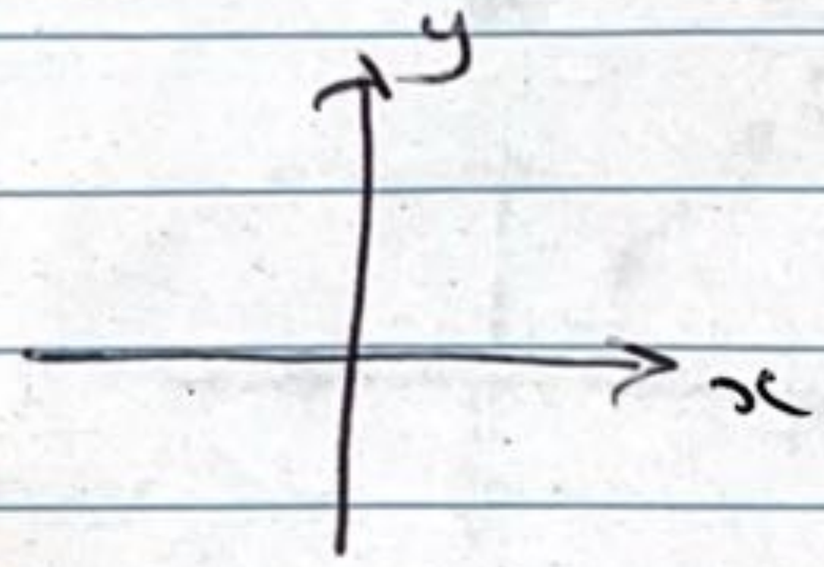
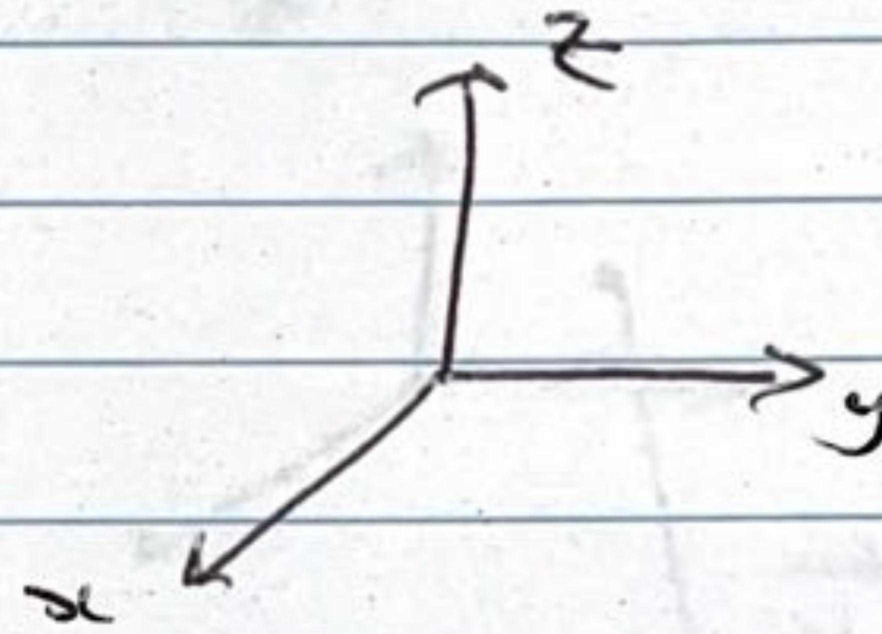
12.1, 12.2 EUCLIDEAN SPACE AND VECTORS

UNIVERSE

THE PLANE, $\mathbb{R}^2$ or SPACE, $\mathbb{R}^3$





POINTS

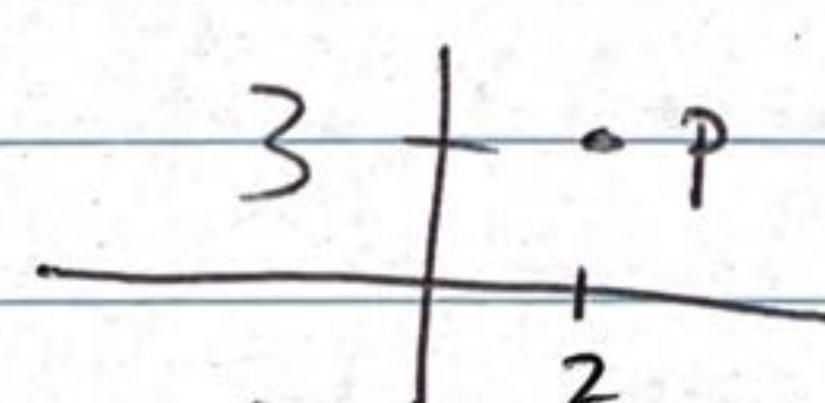
PHYSICS DEF^N: A POINT, p, IS A POSITION IN $\mathbb{R}^n$ ($n=2$ or 3)

COMPUTER SCIENCE (CS) DEF^N:

A POINT, p, IS AN ARRAY (ORDERED LIST) OF n REAL NUMBERS ($n=2$ or 3)

$\mathbb{R}^2$ $\quad p = \begin{pmatrix} x \\ y \end{pmatrix}$

EX $\quad p = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\boxed{CS}$

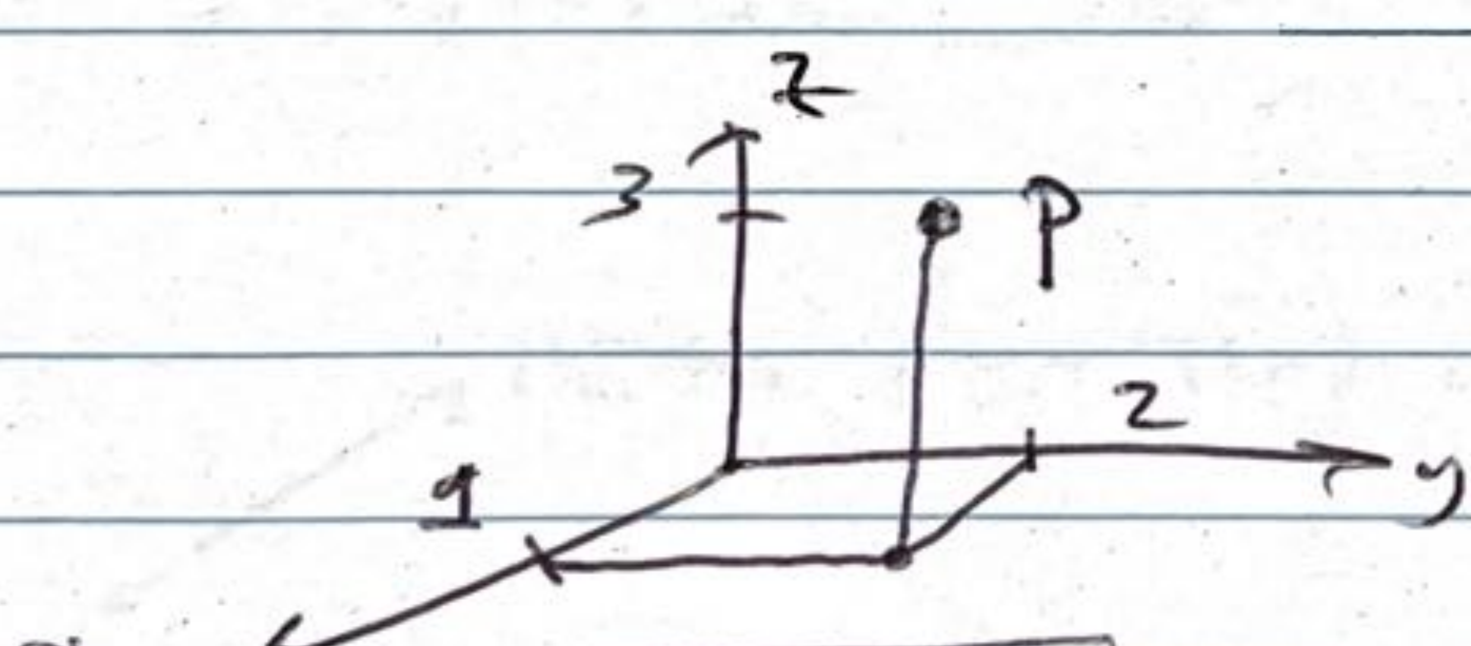


PHYSICS

$\mathbb{R}^3$ $\quad p = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$p = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

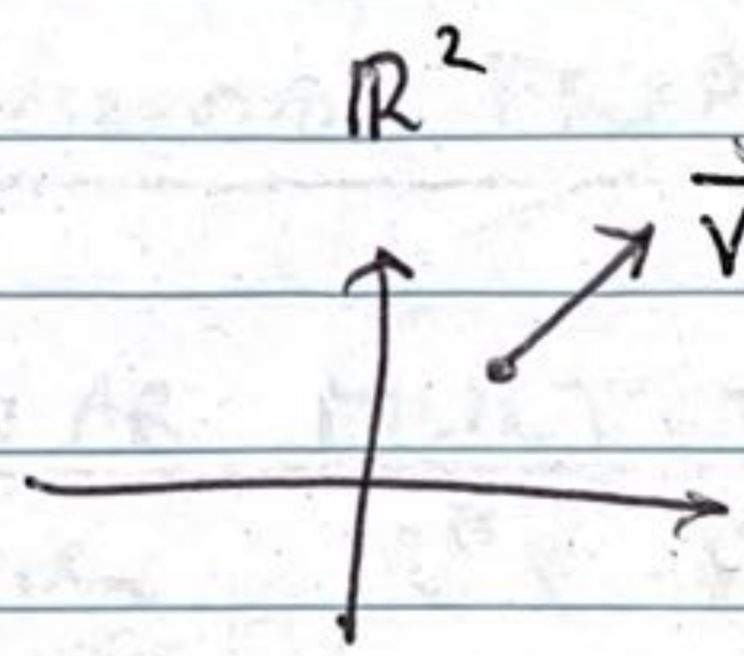
$\boxed{CS}$

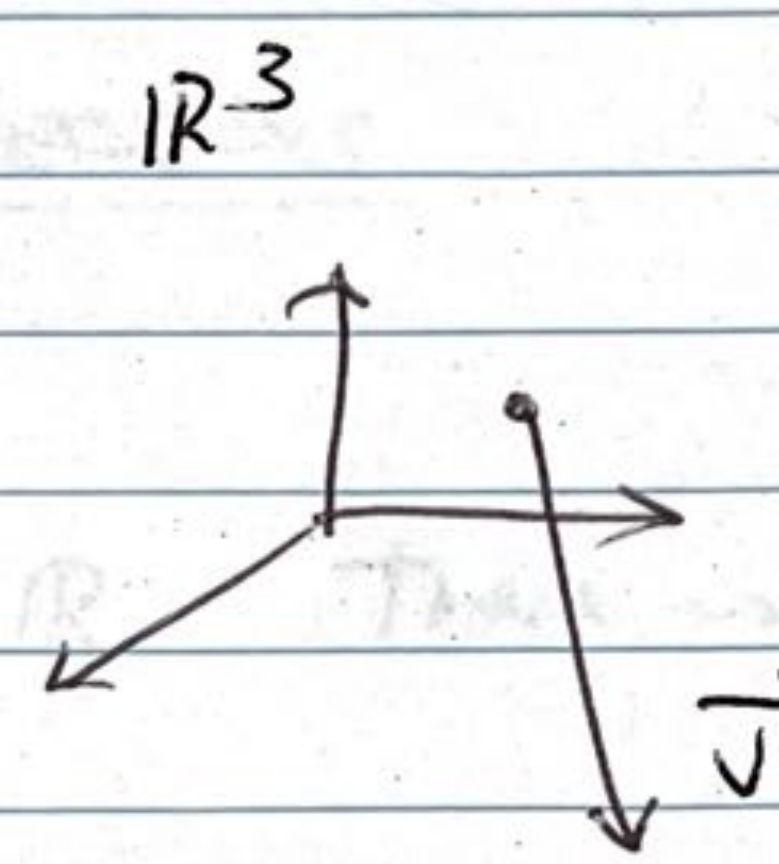


PHYSICS

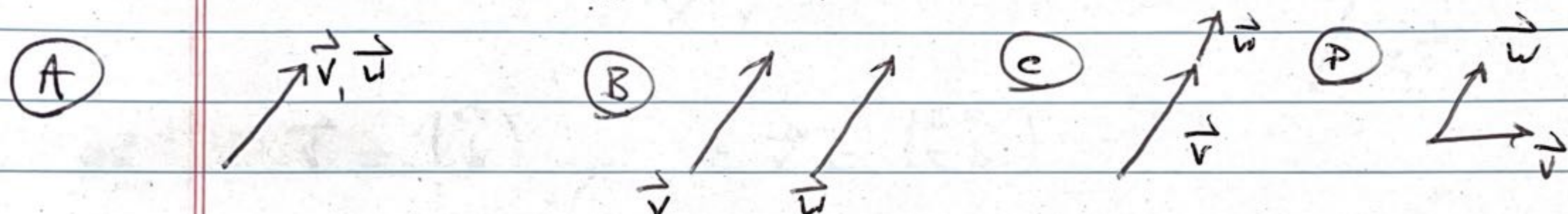
VECTORS

PHYSICS DEF: A VECTOR, $\vec{v}$, is a quantity with MAGNITUDE and DIRECTION





QUESTION Which pairs of vectors are the same?



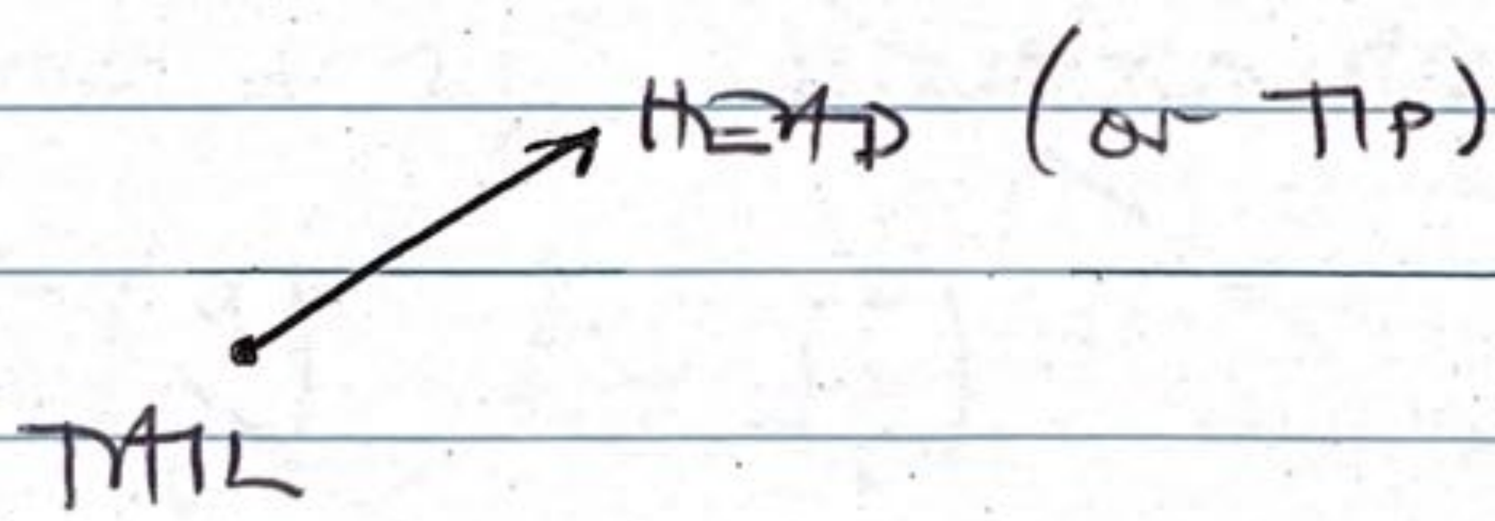
ANSWER A, B : In both cases $\vec{v}, \vec{w}$ have same magnitude + direction, so they are same vector.

In B it does not matter that $\vec{v}, \vec{w}$ have different starting (base) points.

C Different magnitudes $\Rightarrow \vec{v} \neq \vec{w}$

D Different directions $\Rightarrow \vec{v} \neq \vec{w}$

PHYSICS TERMINOLOGY



<u>CS DEFN</u> of Vector is same as CS def^n of point.

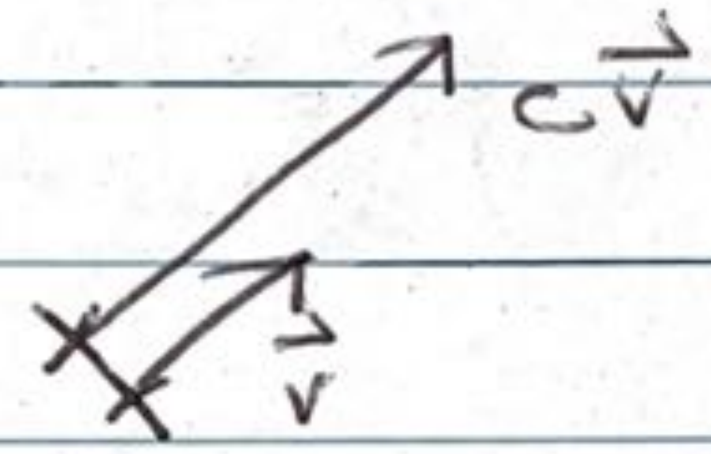
But vectors have special properties:

2 FUNDAMENTAL PROPERTIES OF VECTORS

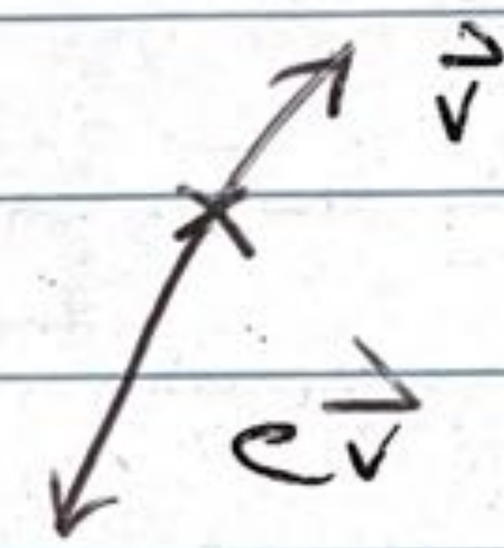
(a) SCALAR MULTIPLICATION

Given $\vec{v} \in \mathbb{R}^n$ and $c \in \mathbb{R}$ there is a vector $c\vec{v} \in \mathbb{R}^n$ with

<u>PHYSICS</u> $\boxed{c>0}$



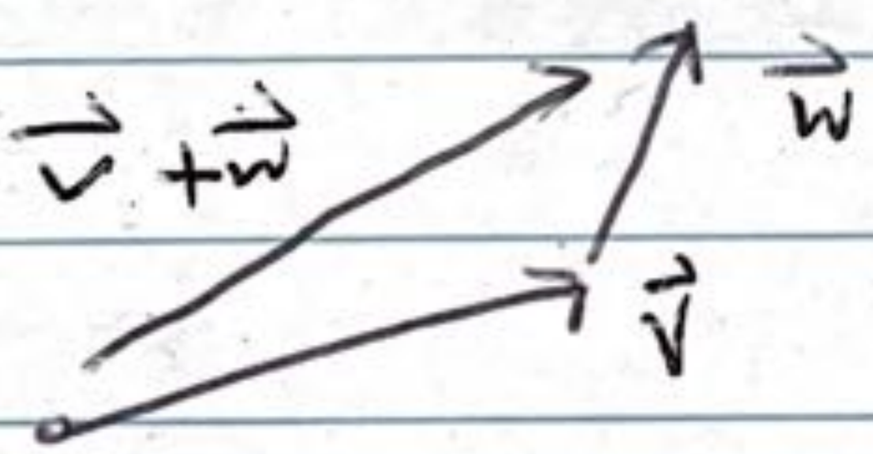
$\boxed{c<0}$



<u>CS</u> $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$, $c\vec{v} = \begin{pmatrix} ca \\ cb \end{pmatrix}$

(b) VECTOR ADDITION

<u>PHYSICS</u>

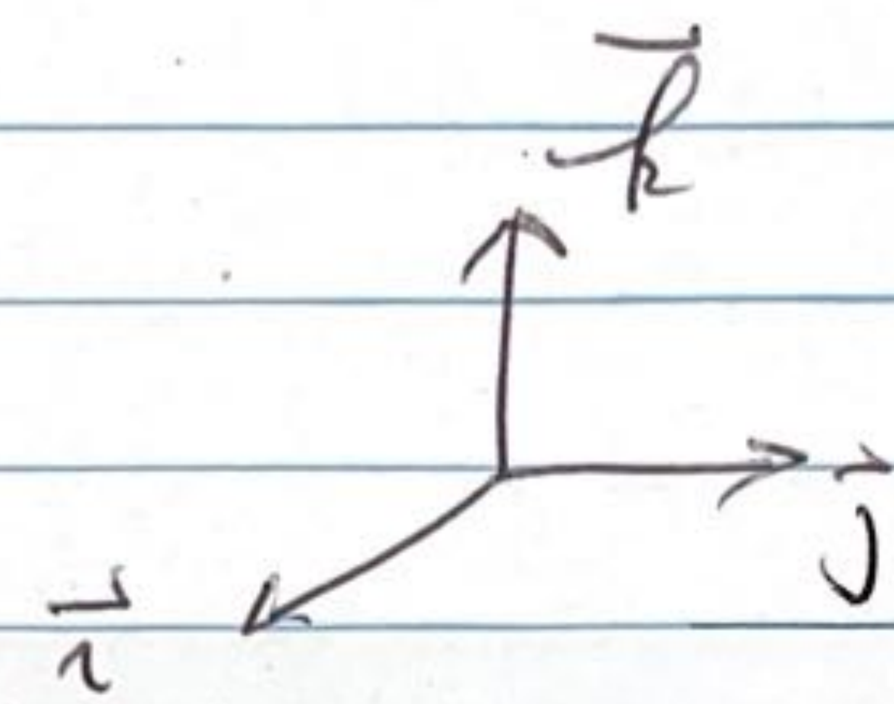


<u>CS</u> IF $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$, $\vec{w} = \begin{pmatrix} c \\ d \end{pmatrix}$

Then $\vec{v} + \vec{w} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$

SPECIAL VECTORS

1. $\mathbb{R}^3$ $\vec{i}$, $\vec{j}$, $\vec{k}$



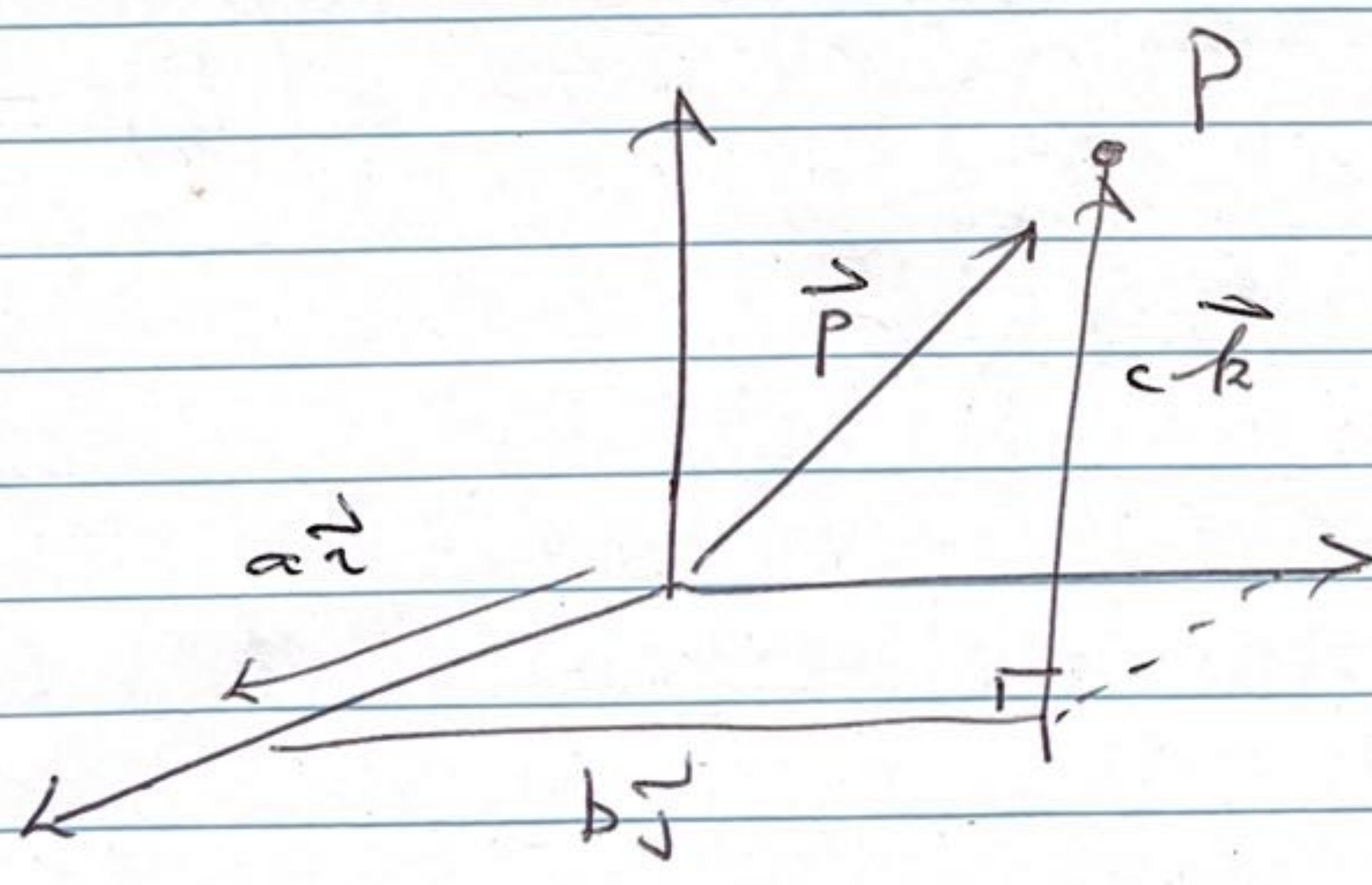
PHYSICS MAGNITUDE = 1
DIRECTED ALONG x, y, z AXES

CS $\vec{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\vec{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\vec{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

[IN $\mathbb{R}^2$ HAVE $\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$]

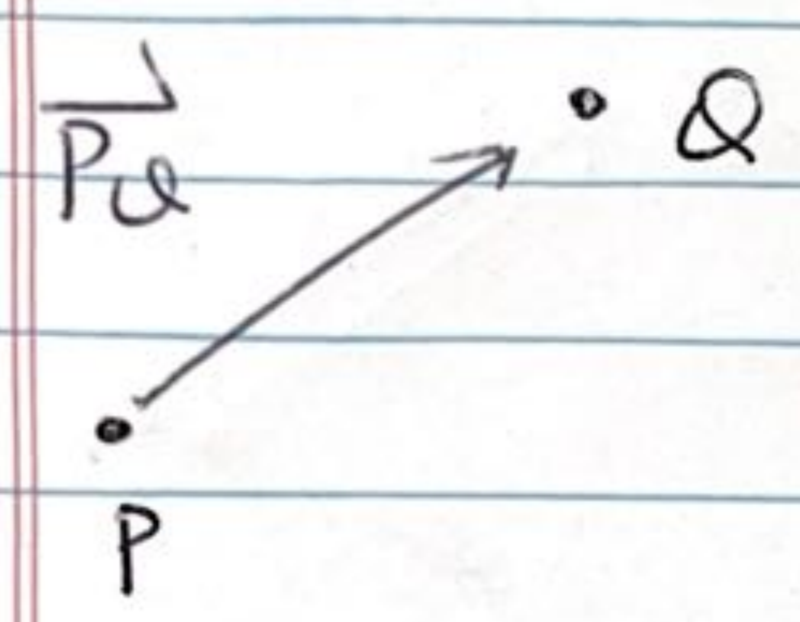
2. THE POSITION VECTOR OF A POINT P
IS THE VECTOR FROM ORIGIN TO P.

IF $P = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ THEN $\vec{p} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a\vec{i} + b\vec{j} + c\vec{k}$.



3. $\overrightarrow{PQ}$ = <u>DISPLACEMENT</u> <u>VECTOR</u> FROM POINT P TO PT Q

<u>PHYSICS</u>



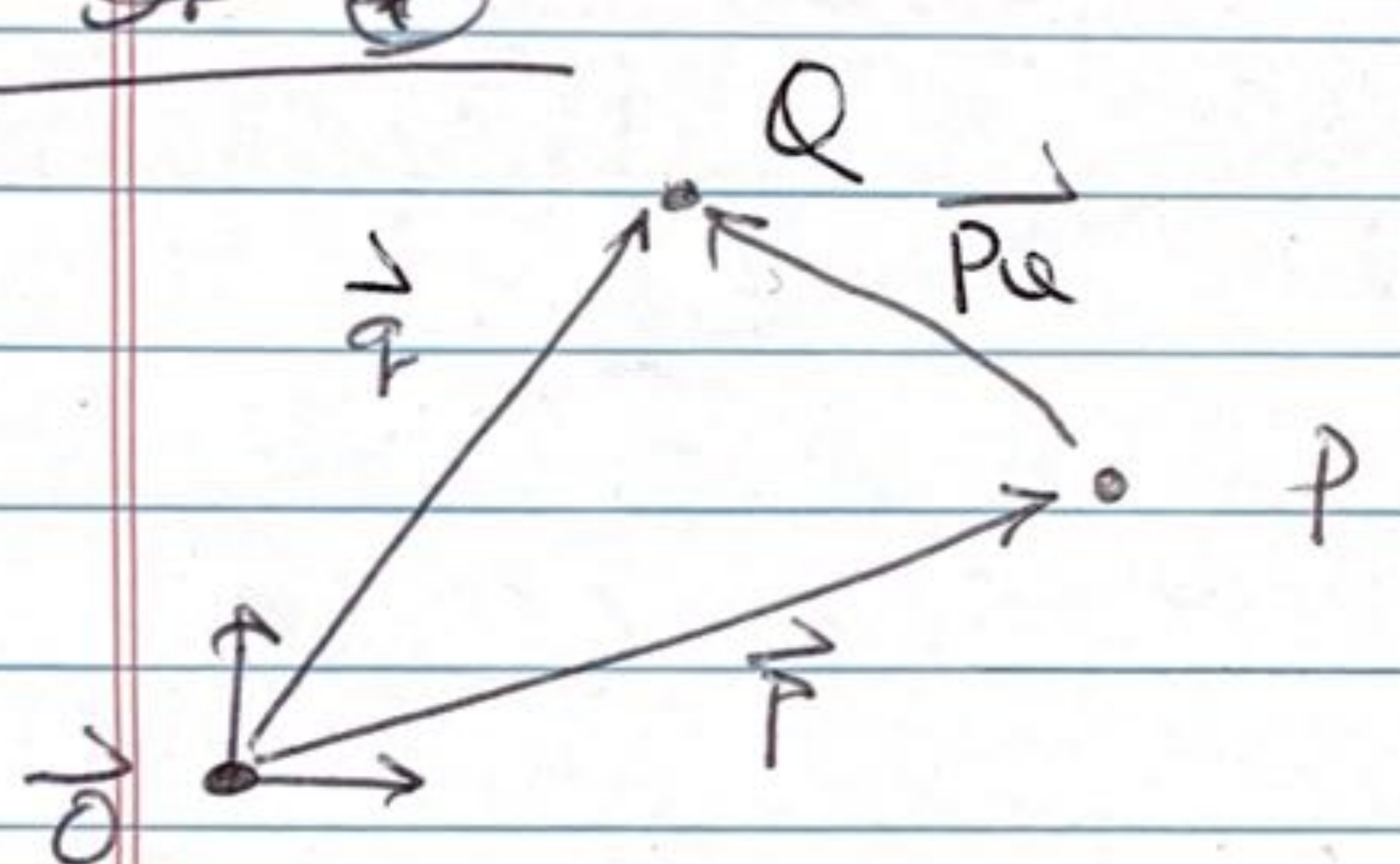
<u>CS</u>

$$\boxed{\overrightarrow{PQ} = Q - P} \quad (*)$$

<u>EX</u> $P = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad Q = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$$\overrightarrow{PQ} = Q - P = \begin{pmatrix} 3 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

<u>PROOF OF (*)</u>



BY Δ LAW OF ADDITION

$$\vec{p} + \overrightarrow{PQ} = \vec{q}$$

So

$$\overrightarrow{PQ} = \vec{q} - \vec{p}$$

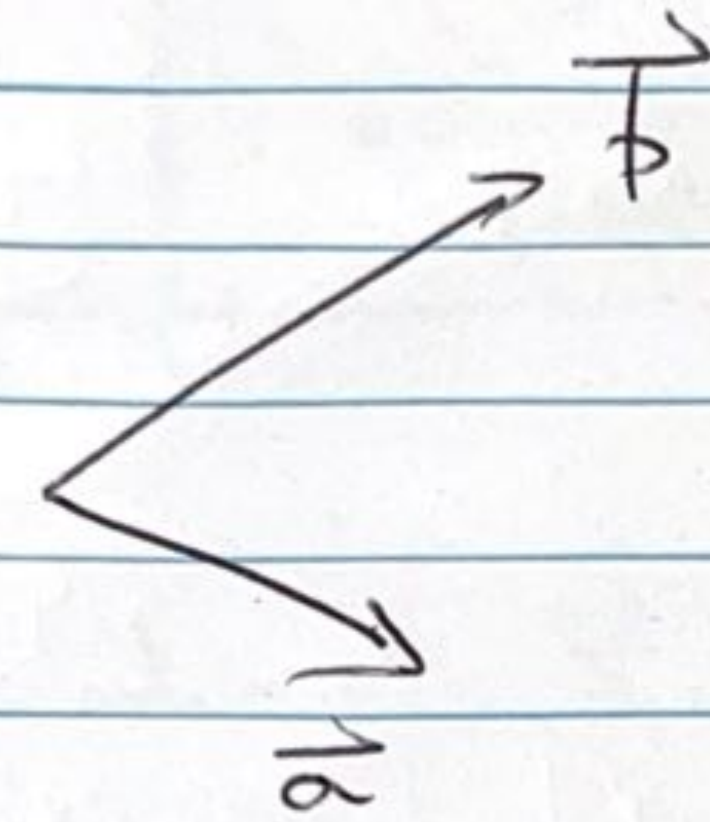
$$\overrightarrow{PQ} = Q - P$$

<u>NOTATION</u>: $|\vec{v}|$ = LENGTH OF VECTOR $\vec{v}$.

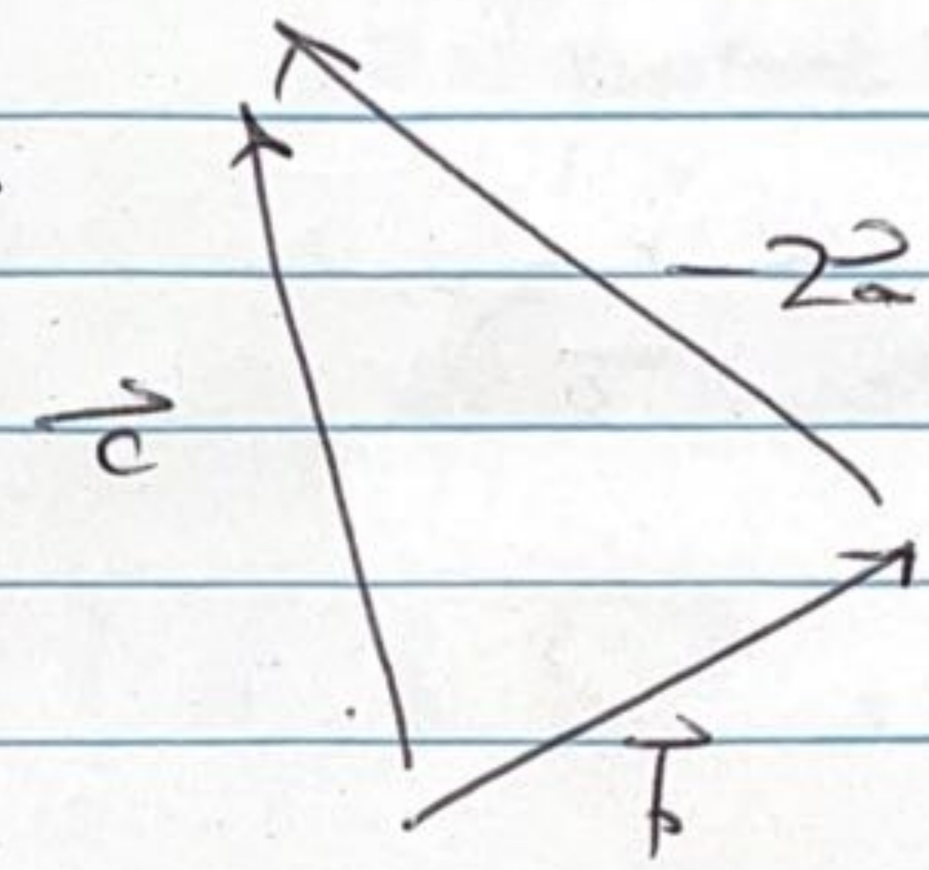
QUESTIONS

USE SCALAR MULTN + VECTOR ADDITION TO SOLVE

1. DRAW $\vec{c} = \vec{b} - 2\vec{a} = \vec{b} + (-2\vec{a})$

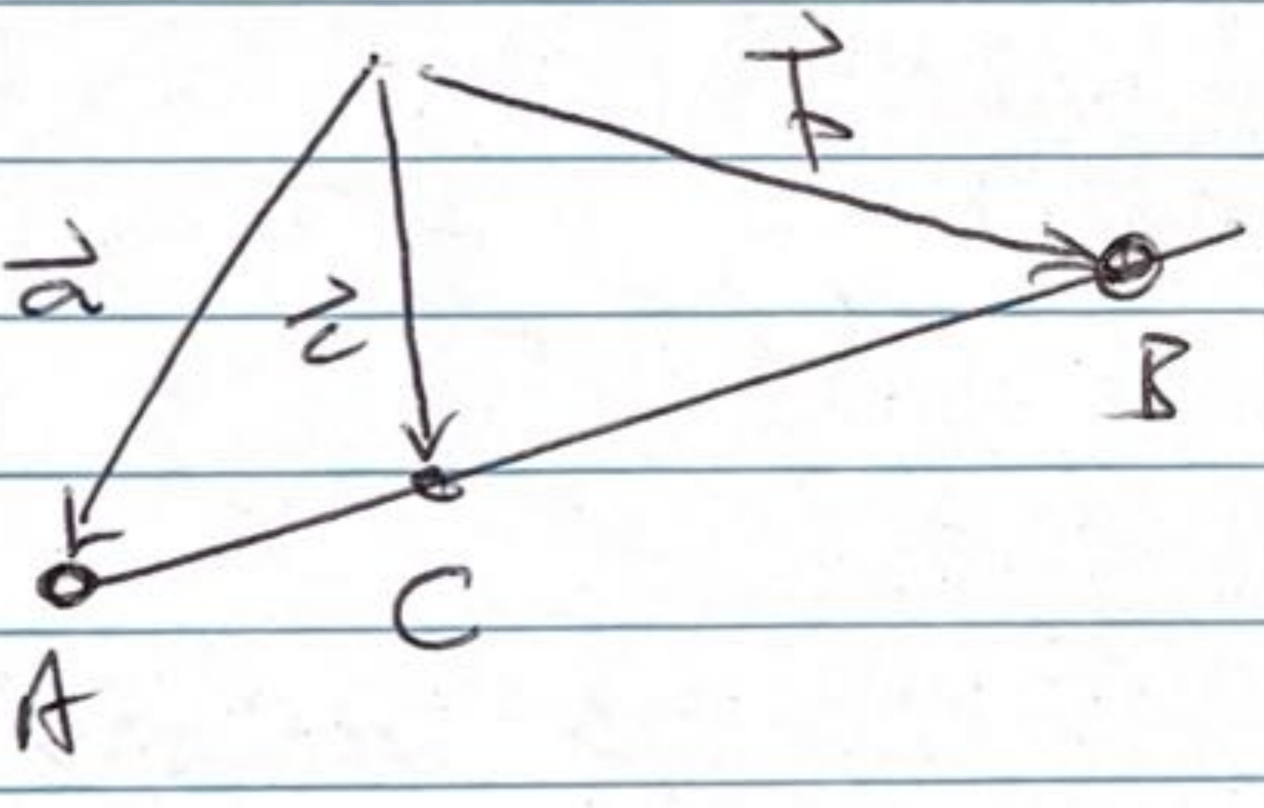


SOLN



2. FIND FORMULA FOR $\vec{c}$ IN TERMS OF $\vec{a}, \vec{b}$
GIVEN

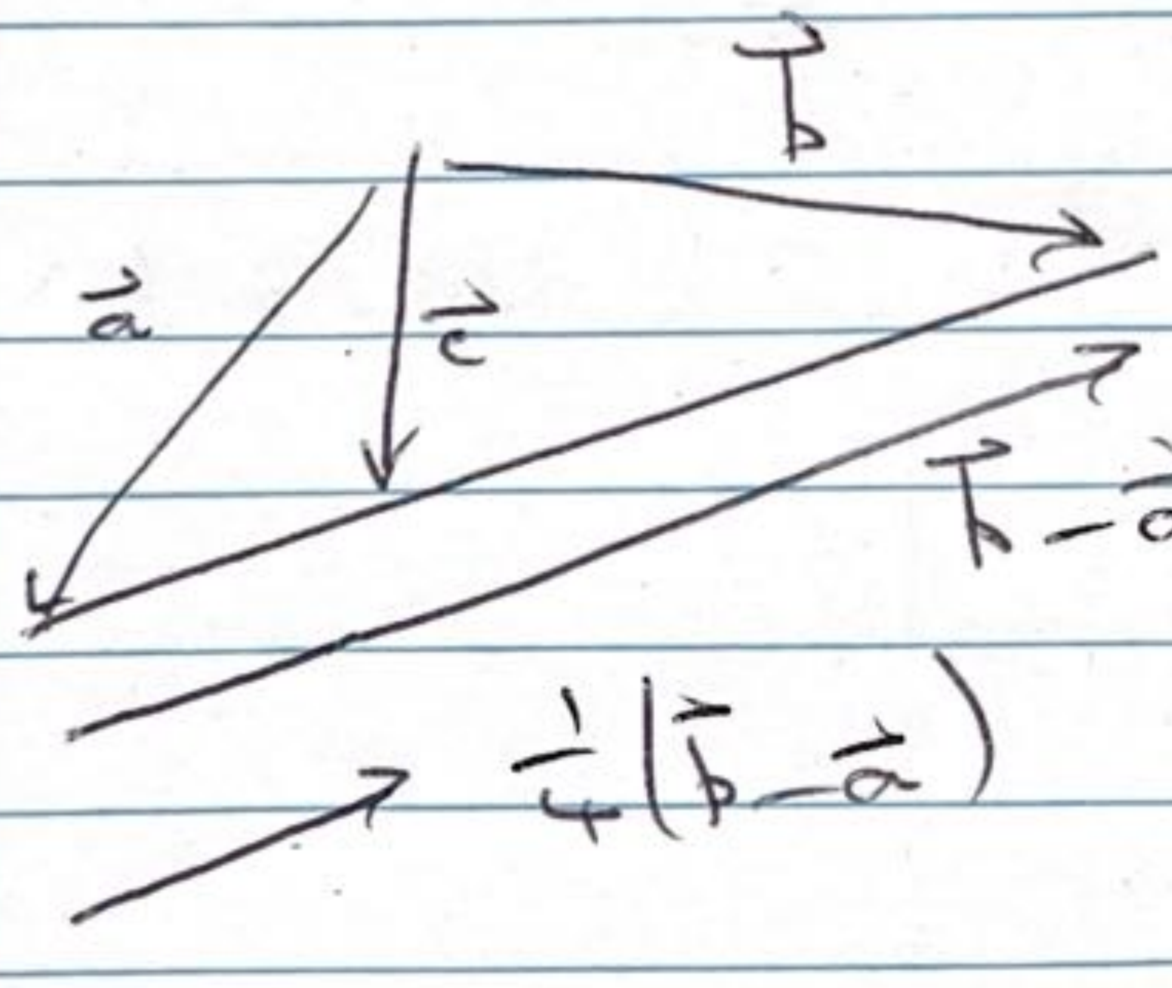
$$|\vec{AC}| = \frac{1}{4}|\vec{AB}|$$



SOLN

$$\vec{c} = \vec{a} + \frac{1}{4}(\vec{b} - \vec{a})$$

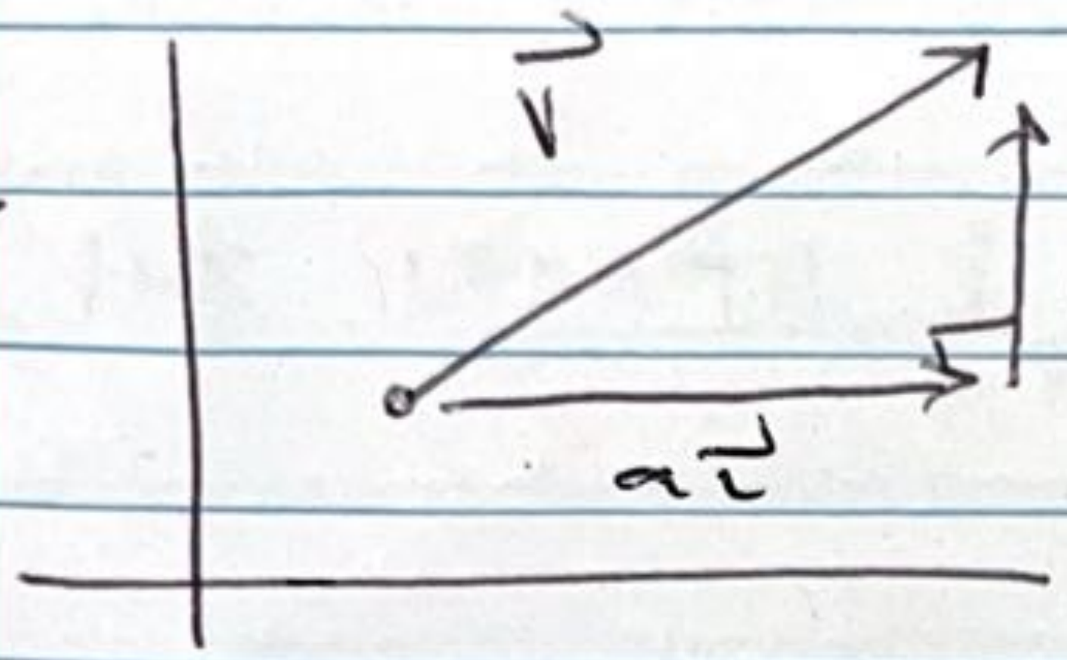
$$\vec{c} = \frac{3}{4}\vec{a} + \frac{1}{4}\vec{b}$$



CONVERTING FROM PHYSICS TO CS DEFN

$\mathbb{R}^2$

PHYSICS



Any vector can be expressed as
$\vec{v} = a\vec{i} + b\vec{j}$
for some $a, b \in \mathbb{R}$

CS So $\vec{v} = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$

CONVERTING FROM CS TO PHYSICS DEFN

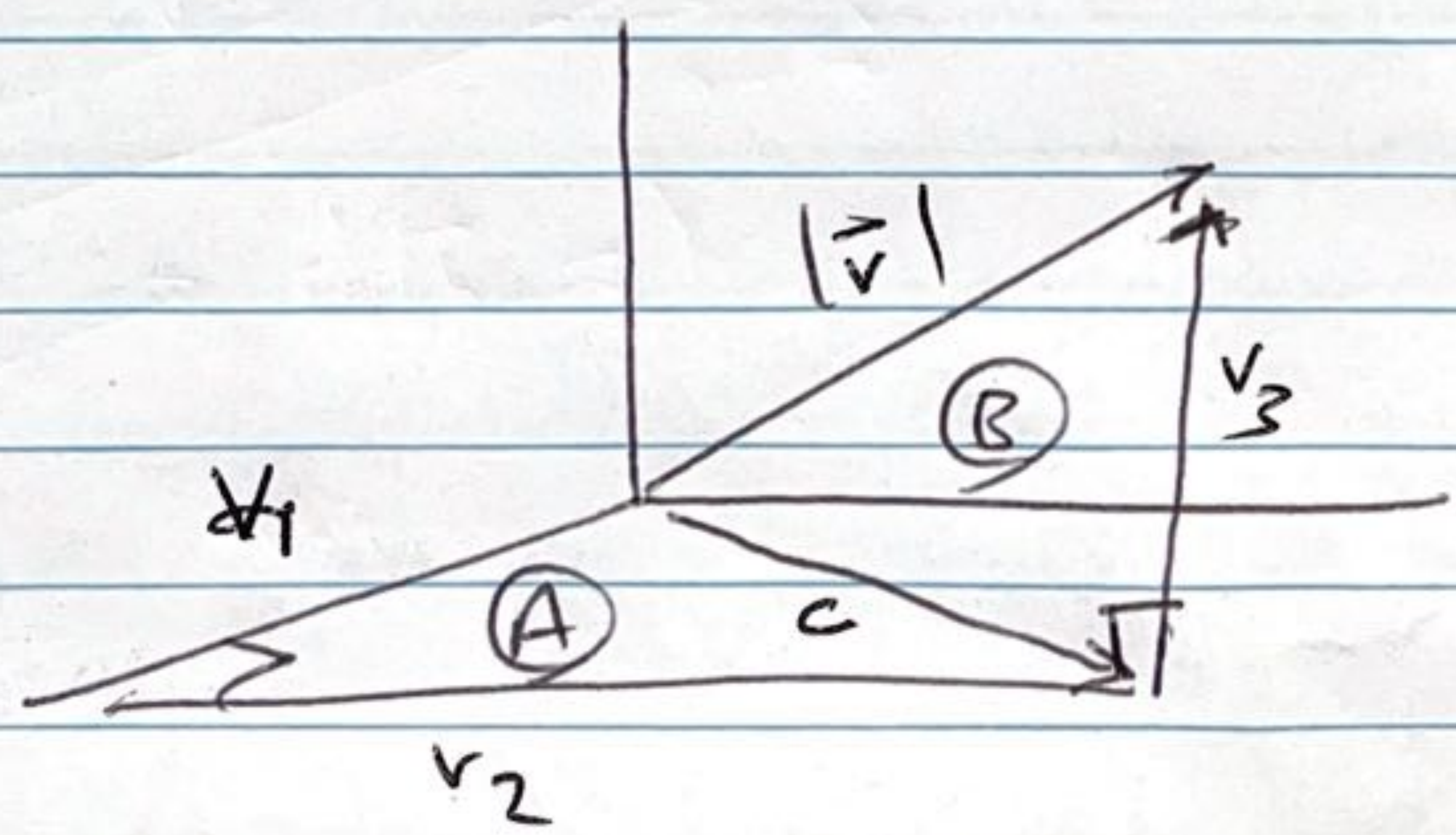
CS Suppose $\vec{v} = \begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$

PHYSICS

(a) MAGNITUDE OF $\vec{v}$ = LENGTH OF $\vec{v}$

$$= |\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

PF PYTHAGORAS x 2 :



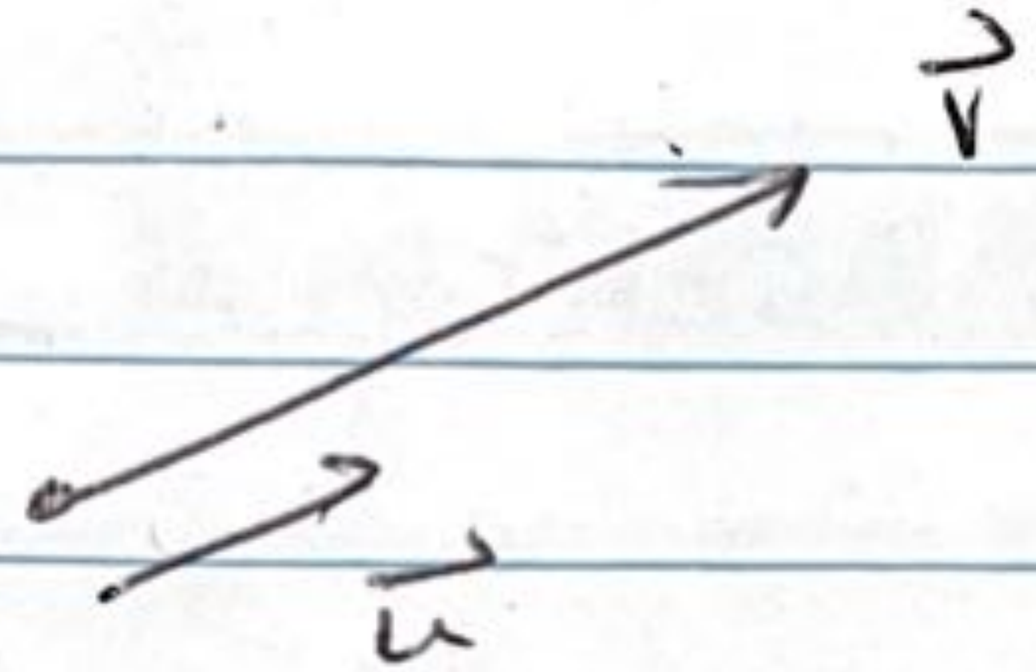
(A) $c^2 = v_1^2 + v_2^2$

(B) $|\vec{v}|^2 = c^2 + v_3^2$

$= v_1^2 + v_2^2 + v_3^2$

<u>DIRECTION OF $\vec{v}$</u> IS $\vec{u} = \dfrac{\vec{v}}{|\vec{v}|}$

$\vec{u}$ HAS LENGTH 1, $\parallel$ to $\vec{v}$, PTS SAME DIRN



BEAUTIFUL FORMULA

$$\vec{v} = |\vec{v}| \; \frac{\vec{v}}{|\vec{v}|}$$

VECTOR = LENGTH × DIRECTION

INTRODUCTION TO SURFACES IN $\mathbb{R}^3$

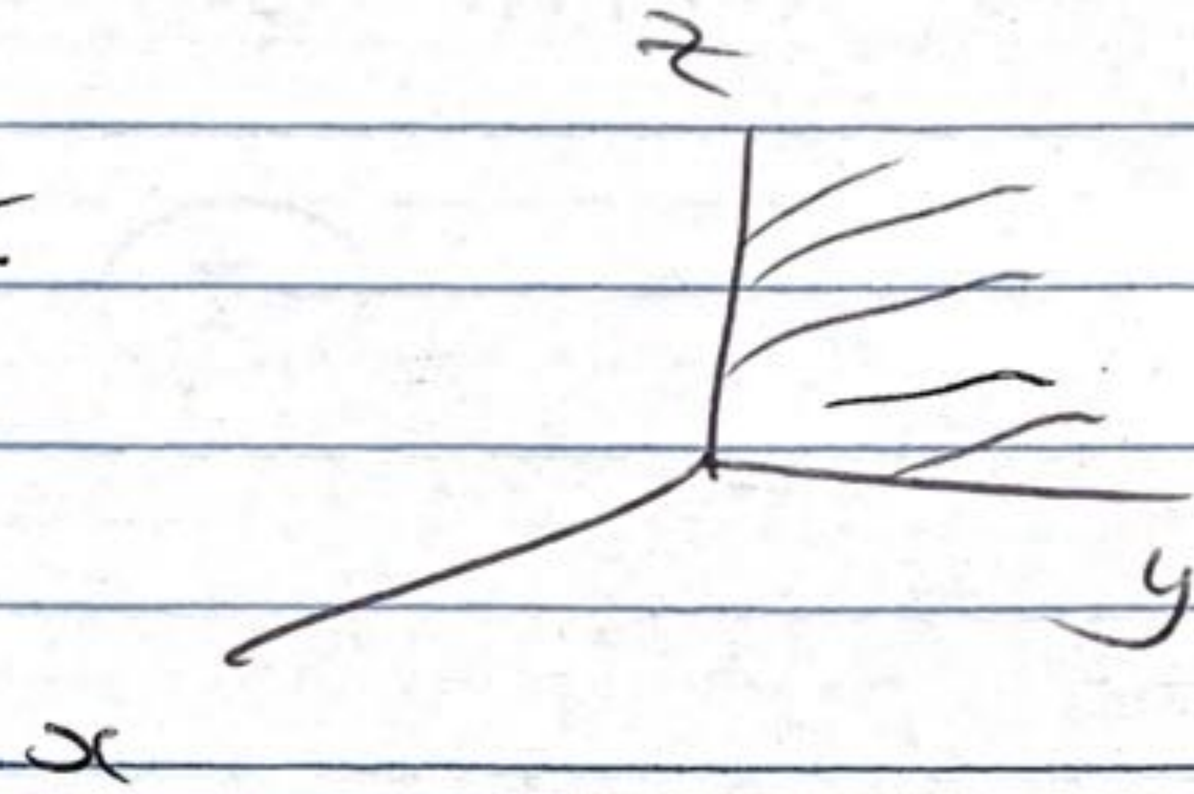
THE SET OF POINTS $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ SATISFYING

ONE EQUATION IN THE 3 UNKNOWNS x, y, z

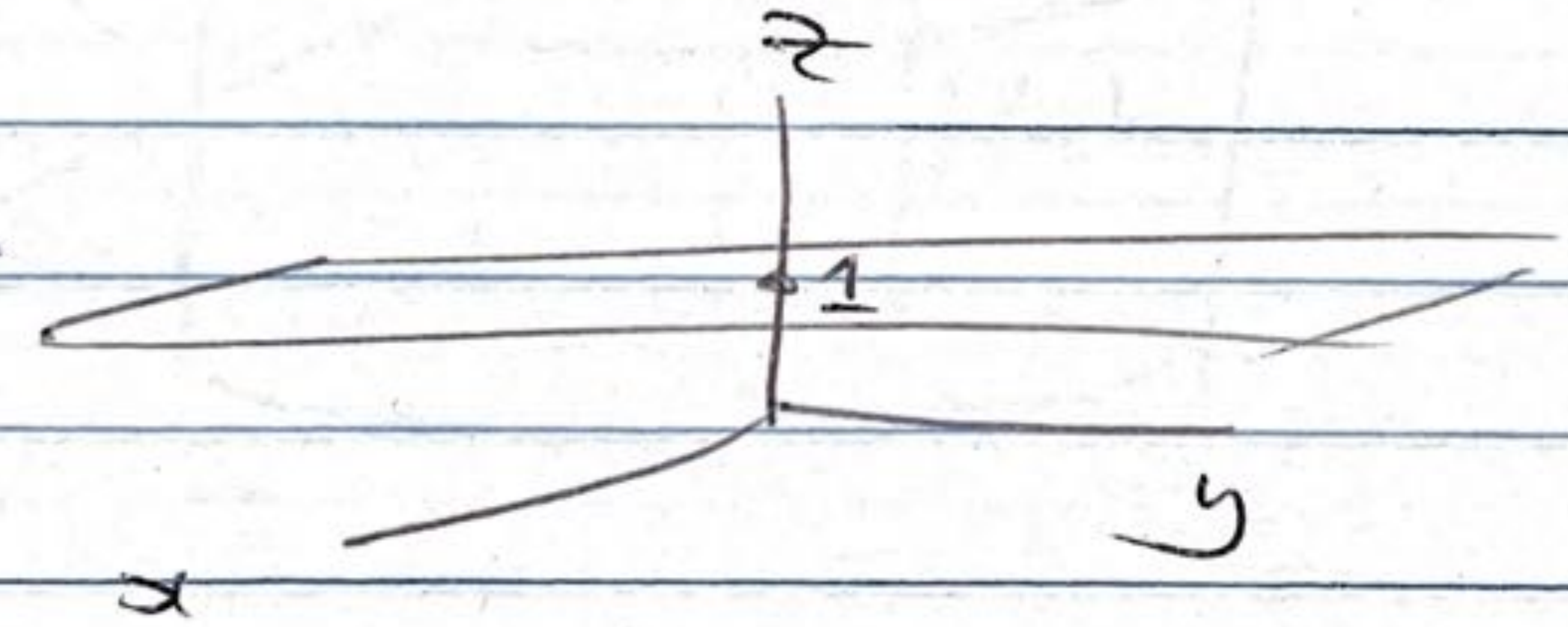
FORMS A 2-DIMENSIONAL (2D) SURFACE IN $\mathbb{R}^3$.

EXS

① $x = 0$ IS yz-PLANE



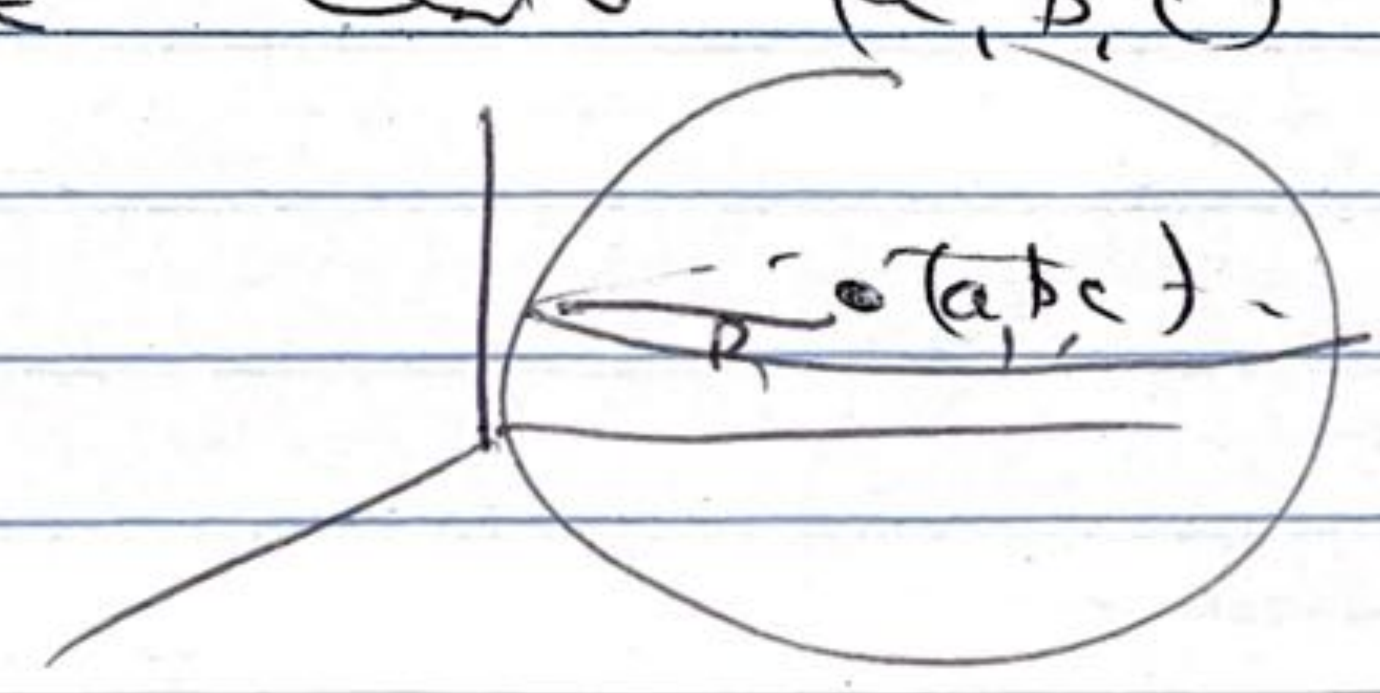
② $z = 1$ IS HORIZONTAL PLANE // TO xy-PLANE



③ $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^3$

IS SET OF POINTS (xyz) whose distance from (a,b,c) is R.

SPHERE center (a,b,c) radius R



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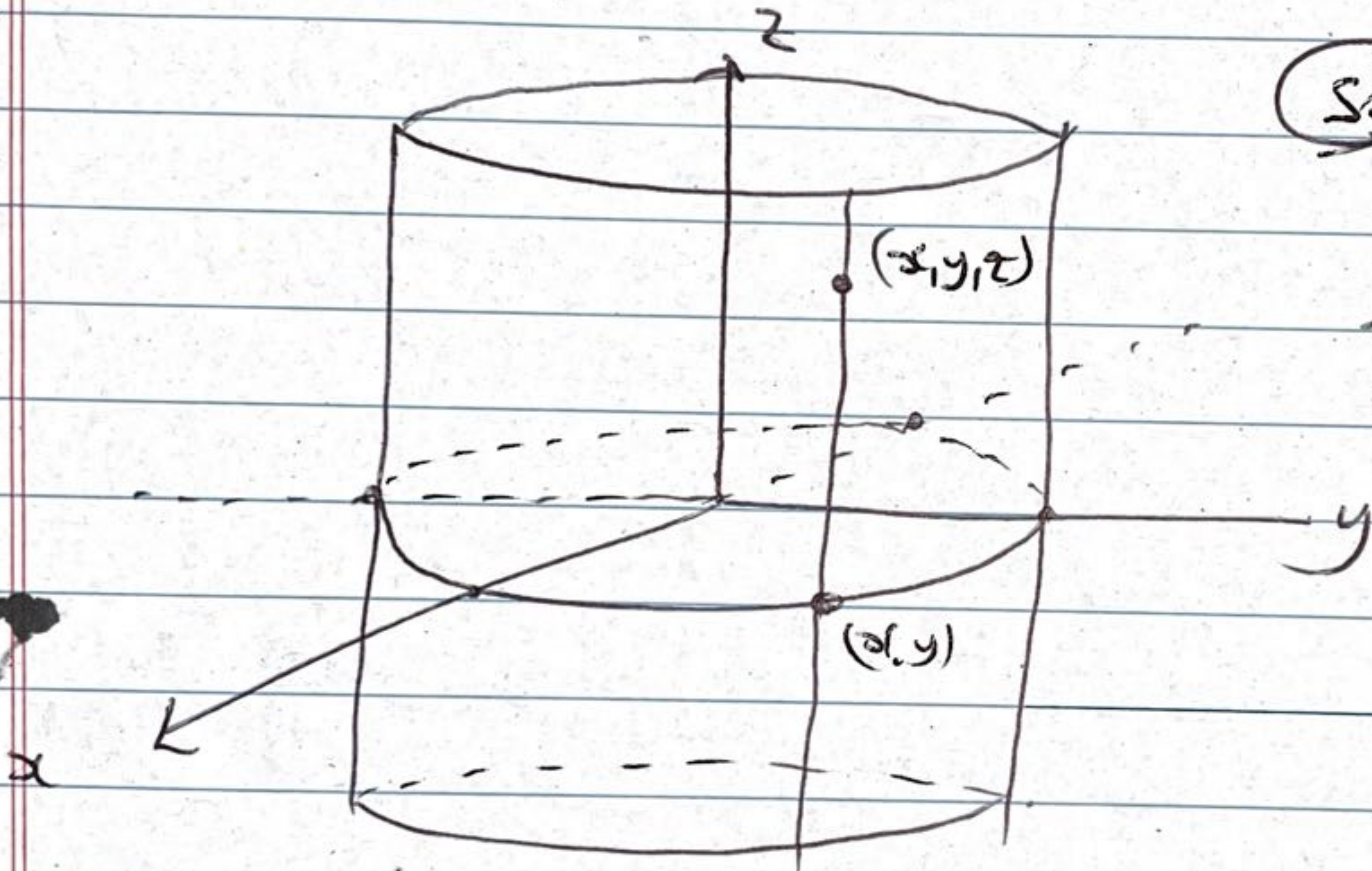
$\underline{\underline{Q}}$ (4) What is surface, S, consisting of all pts $(x, y, z) \in \mathbb{R}^3$ with $x^2 + y^2 = 1$?

$\underline{\underline{A}}$ $(x, y, z) \in S$ precisely when shadow (x, y) in plane $z = 0$ lies on circle $x^2 + y^2 = 1$



So

S is curved surface of ∞ cylinder