Math 2415

Paper Homework #4

1. **[12.6: Cylinders and Quadric Surfaces]** For each part, first decide which of the three methods we discussed in the lecture and problem sections will *most easily* enable you to sketch the surface. These are the generalized cylinder method, the surface of revolution method, and the method of slices (traces) for general quadric surfaces. Then apply that method to sketch the surface. You will be graded on how well you demonstrate your understanding of how to apply the method rather than for how perfect your final sketch looks.

   (a) $z^2 = y^2 + 4$

   (b) $z^2 - x^2 - y^2 = 4$

   (c) $\frac{y^2}{9} + \frac{z^2}{4} = 1 + \frac{x^2}{16}$ **Hint:** Choose several slices that are ellipses and only two that are hyperbolae.

2. **[12.6: Cylinders and Quadric Surfaces]** Make a labelled sketch of the traces (slices) of the surface

$$2x^2 - y^2 - 4z^2 = 8$$

   in the planes $y = 0$, $z = 0$, and $x = k$ for $k = 0, \pm 1, \pm 2, \pm 3$. Then sketch the surface.

3. **[13.1: Parametrized Curves]** Use a method from the Problem Section on 13.1 to parametrize the curve obtained by intersecting the cylinder $x^2+y^2 = 1$ and the saddle surface $z = y^2-x^2$. [**Hint:** You may find that a double angle formula from trigonometry is helpful.]

   Sketch the graph of $z = z(t)$ on a piece of paper so that the t-axis takes up one entire edge of the paper. Wrap the paper into a cylinder in such a way that the graph you sketched is the curve of intersection of the cylinder and saddle surface. Include your sketch of the graph of $z = z(t)$ in your homework solutions.

4. **[13.1: Parametrized Curves]** Parametrize the curve obtained by intersecting the cylinder $y^2 + z^2 = 4$ and the plane $y = x$. Make a single sketch showing both surfaces and their curve of intersection. Finally, find an equation of a second cylinder on which this curve lies.

5. **[13.1: Parametrized Curves]** Show that the curve with parametrization $x = t\sin 2t$, $y = t$, $z = t\cos 2t$ for $\pi/2 \le t \le 2\pi$ lies on the cone $y = \sqrt{x^2 + z^2}$. Sketch the cone and the curve.