Math 2415

Paper Homework #3

1. **[12.5B: Planes]** Let $A = (2, 0, 1)$, $B = (3, 1, 0)$ and $C = (4, 3, 2)$.
   (a) Find the level set equation of the plane, $\mathcal{P}$, containing the points A, B, and C.
   (b) Check that A, B, and C each satisfy the equation you derived in (a).
   (c) Find a parametrization, $(x, y, z) = \mathbf{r}(s, t)$, of the plane, $\mathcal{P}$, containing A, B, and C.
   (d) For each of the three points, A, B, and C, find values of the parameters (s, t) in the parameterization you found in (c).
   (e) Let $\mathcal{L}$ be the line passing through the point $(-1, 0, 2)$ that is parallel to the vector $(1, 2, 3)$. Find the point of intersection of this line with the plane, $\mathcal{P}$.

2. **[12.5B: Planes]** Find a non-zero vector that is parallel to both of the planes $x + y + 2z = 5$ and $y - 6z = 7$.

3. **[15.7A: Cylindrical Coordinates]** Consider the following points, curves, surfaces, and solids
   (i) The surface $r = 2$.
   (ii) The curve where $r = 2$ and $z = 3$.
   (iii) The curve where $r = 2$ and $\theta = \pi/4$.
   (iv) The point $(r, \theta, z) = (2, \pi/4, 3)$.
   (v) The solid where $r \leq 2$, $0 \leq \theta \leq \pi/4$ and $0 \leq z \leq 3$.

   Now do the following problems:

   (a) Sketch (i)-(iv) *altogether in one plot, with labels*.
   (b) Sketch (v).
   (c) Convert the equation $r = 2$ to an equation involving spherical coordinates, (ρ, θ, ϕ).
   (d) Parametrize the line where $r = 2$ and $\theta = \pi/4$.
   (e) Find the rectangular and spherical coordinates of the point in (iv).

4. **[15.8A: Spherical Coordinates]**
   (a) Sketch the surface whose equation is given by $\phi = 5\pi/6$
   (b) Convert the equation $\phi = 5\pi/6$ to an equation involving cylindrical coordinates (r, θ, z).
   (c) Convert the equation $x^2 + y^2 - 3z^2 = 1$ to an equation involving spherical coordinates, (ρ, θ, ϕ). (The answer is not pretty, but that's OK.)
   (d) Sketch the solid described by the inequalities $1 \leq \rho \leq 4$, $\pi/4 \leq \phi \leq \pi/2$.