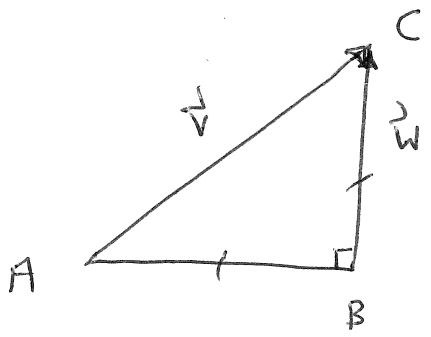
Math 2415

Paper Homework #2

1. **[12.3, Projections]**

   (a) Let $P = (2, 0, 1)$, $Q = (3, 1, 0)$ and $R = (4, 3, 5)$. Calculate the projection of the vector $\overrightarrow{PR}$ onto the vector $\overrightarrow{PQ}$.

   (b) The triangle ABC in the figure below is an isoceles triangle for which the length of the hypotenuse is 1. Let $\mathbf{v} = \overrightarrow{AC}$ and $\mathbf{w} = \overrightarrow{BC}$. Calculate (a) $\mathbf{v} \cdot \mathbf{w}$, (b) the scalar projection of $\mathbf{w}$ onto $\mathbf{v}$, and (c) find a scalar a so that the vector projection of $\mathbf{v}$ onto $\mathbf{w}$ is $a\mathbf{w}$.



2. **[12.4, Cross Products]**

   Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} - 5\mathbf{j}$ and $\mathbf{c} = \mathbf{j} + 2\mathbf{k}$.

   (a) Find the length of $\mathbf{a}$.

   (b) Find a unit vector that is orthogonal to both $\mathbf{a}$ and $\mathbf{c}$.

   (c) Calculate the area of the parallelogram determined by the vectors $\mathbf{a}$ and $\mathbf{c}$.

   (d) Calculate the volume of the parallelipiped determined by the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

3. **[12.4, Cross Products]** Consider the triangle with vertices $(10, 7, 13)$, $(1, 2, 3)$, $(4, 1, 2)$.

   (a) Find a point $\mathbf{p}$ and two vectors $\mathbf{u}$ and $\mathbf{v}$ so that that the triangle has $\mathbf{p}$ as a vertex and the vectors $\mathbf{u}$ and $\mathbf{v}$ as edges.

   (b) Use your answer to 3a to find the area of the triangle.

4. **[12.5A, Lines]**

   (a) Find a vector parametrization for the line, $\mathcal{L}$, passing through the points $P = (1, 2, 3)$ and $Q = (5, -6, 17)$.

(b) Which of the points are on the line $\mathcal{L}$? Which are on the line and are between P and Q? Why?

i. $(4, 5, 2)$,

ii. $(3, -2, 10)$,

iii. $(-1, 6, -4)$.

(c) Determine whether the line, $\mathcal{L}$,

i. intersects the xy-plane,

ii. intersects with the z-axis.