Math 2415

Paper Homework #12

1. **15.7, Triple Integrals in Cylindrical Coordinates**

   (a) Sketch the solid region, E, in the first octant that is bounded by the cylinder $y^2 + z^2 = 16$ and the plane $x + y = 4$. Use a triple integral in cylindrical coordinates to find $\iiint_E (y^2 + z^2)\, dV$.

   (b) Find $\iiint_E \sqrt{x^2 + y^2}\, dV$ where E is the solid region above the xy-plane, below the paraboloid $z = 8 - x^2 - y^2$ and *outside* the cylinder $x^2 + y^2 = 4$. Sketch the solid E.

   (c) Let E be the solid region that lies above the xy-plane and that is bounded by the surfaces $z^2 - x^2 - y^2 = 1$ and $x^2 + y^2 + z^2 = 4$. Calculate the integral $\iiint_E z\, dV$.

2. **15.8, Triple Integrals in Spherical Coordinates**

   (a) Find $\iiint z^2\, dV$ where E is the solid region that is inside the sphere $x^2 + y^2 + z^2 = 9$ and below the cone $z = \sqrt{x^2 + y^2}$.

   (b) Find the volume of the region inside the ball $x^2 + y^2 + z^2 \leq 9$ that lies between the planes $y = 0$ and $y = \sqrt{3}x$ in the first octant.

   (c) Let R be a non-zero constant. Find $\iiint_E (x^2 + y^2 + z^2)xyz\, dV$ where E is

      i. The ball $x^2 + y^2 + z^2 \leq R^2$
      ii. The half-ball $x^2 + y^2 + z^2 \leq R^2$, with $z \geq 0$
      iii. The one-eighth ball $x^2 + y^2 + z^2 \leq R^2$ with $x \geq 0$, $y \geq 0$, and $z \geq 0$.

   Your answer should be given in terms of R. **Hint:** You can use symmetry to simplify the calculation for at least one of these regions.

3. **15.9, Change of Variables Theorem:**

   (a) Evaluate $\iint_R (x - y)^2 e^{x+y}\, dx dy$ where R is the parallelogram bounded by $x - y = 1$, $x - y = 3$, $x + y = -2$ and $x + y = 1$.

   (b) Use the change of variables $u = y/x^2$, $v = x/y^2$ to find the area of the region in the first quadrant that is bounded by the curves $y = x^2$, $y = 4x^2$, $x = y^2$ and $x = 3y^2$.