LAST NAME:							FIRST NAME:						
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## MATH 4362 (Spring 2018), Midterm Exam Two, (Zweck)

**Instructions:** This 75 minute exam is worth 75 points. No books or notes! Show all work and give **complete explanations**. Don't spend too much time on any one problem.

Throughout this exam  $\chi_{[a,b]}$  is the function defined by

$$\chi_{[a,b]}(x) = \begin{cases} 1 & \text{if } a \leq x \leq b, \\ 0 & \text{otherwise.} \end{cases}$$

(1) [12 pts] Prove that  $\{\phi_k(x) = e^{ikx} \mid k = 0, \pm 1, \pm 2, \cdots\}$  is an orthonormal set of functions on  $[-\pi, \pi]$  with respect to the  $L^2$ -inner product.

(2) [12 pts] Find a formula for the solution u=u(t,x) of the PDE initial-value problem

$$u_{tt} = \frac{1}{4}u_{xx}$$

$$u(0, x) = \chi_{[-1,1]}(x)$$

$$u_t(0, x) = 0.$$

Sketch the solution at t = 1 and at t = 4.

(3) [12 pts] Find a formula for the solution u=u(t,x) of the PDE initial-value problem

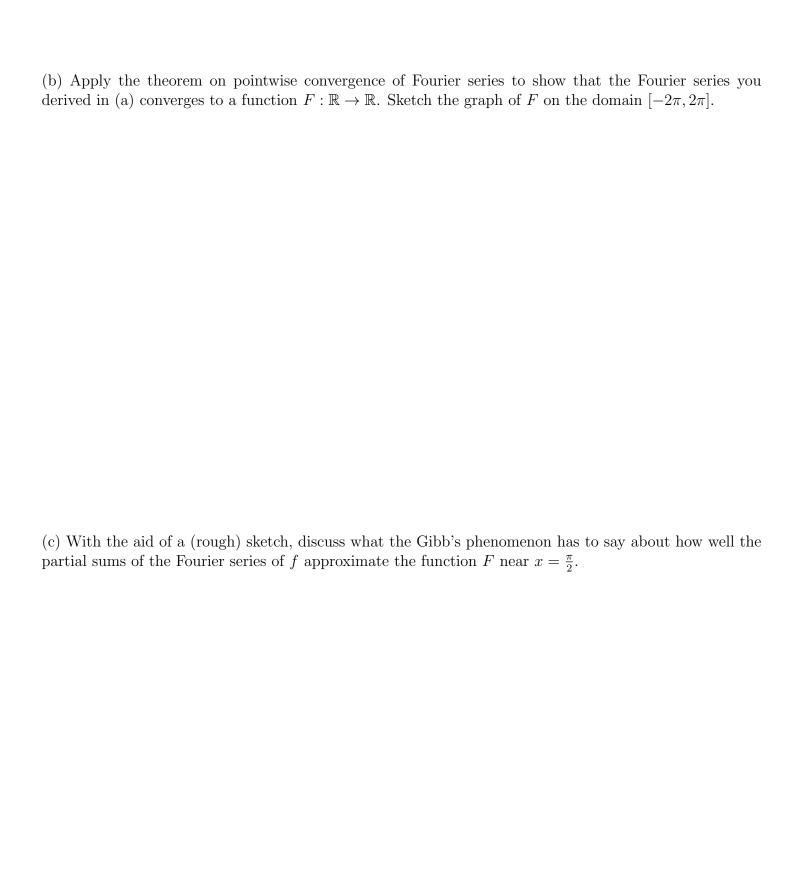
$$u_{tt} = u_{xx}$$

$$u(0, x) = 0$$

$$u_t(0, x) = \cos(x).$$

Sketch the solution at  $t = \frac{\pi}{2}$  and  $t = \frac{3\pi}{2}$ .

- (4) [18 pts] Let  $f: [-\pi, \pi] \to \mathbb{R}$  be defined by  $f(x) = \chi_{[-\pi/2, \pi/2]}(x)$ .
- (a) Calculate the Fourier series of f.



(5) [12 pts] The function  $f: [-\pi, \pi] \to \mathbb{R}$  given by  $f(x) = x^2$  has Fourier series

$$x^{2} \sim \frac{2\pi^{2}}{3} + \sum_{k=1}^{\infty} \frac{4(-1)^{k}}{k^{2}} \cos(kx).$$
 (1)

(a) Apply the theorem on the differentiation of Fourier Series to show that the Fourier series (1) can be differentiated term-by-term.

(b) What happens when you differentiate the Fourier series (1) term-by-term a second time? In particular, does the theorem on the differentiation apply to the Fourier series of the function g(x) = f'(x) = 2x?

(6) [9 pts] Suppose that u=u(t,x) satisfies the inhomogeneous wave equation

$$u_{tt} - 9u_{xx} = F(t, x)$$
$$u(0, x) = 0$$
$$u_t(0, x) = 0,$$

where  $F(t,x) = \chi_{[0,1]}(x)$  for all t > 0. Use the concept of the domain of dependence to show that u(2,8) = 0. Then find all x so that u(2,x) = 0.