LAST NAME:	FIRST NAME:

1 /15	2 /15	3 /15	4 /10	5 /10	6 /10	T /75

MATH 4362 (Spring 2018)

Midterm Exam One
(Zweck)

Instructions: This 75 hour exam is worth 75 points. No books or notes! Show all work and give **complete explanations**. Don't spend too much time on any one problem.

(1) [15 pts] Complete the table.

PDE	Order	Equilibrium or Dynamic	Linear or Nonlinear	Homogeneous or Inhomogeneous	Name
$u_t - 4u_{xx} = 0$					
$u_{tt} - 16u_{xx} = \sin x \cos t$					
$u_t + uu_x = 0$					
$u_{xx} + u_{yy} = f(x, y)$					
$u_t + e^{-x}u_x + u = 0$					

(2) [15 pts] Solve the initial value problem for $u = u(t, x)$ given by

$$\begin{aligned} u_t - 4u_x + 3u &= 0, \\ u(0, x) &= e^{-x^2}. \end{aligned}$$

(3) [15 pts] Consider the PDE for $u = u(t,x)$ given by $u_t + x^2 u_x = 0$.

(a) By solving the ODE for the characteristics show that the characteristic curve that goes through the point (t_1, x_1) is given by

$$x = x(t) = \frac{x_1}{1 + x_1(t_1 - t)}. \tag{1}$$

(b) Show that if $x_1 > 0$ then the characteristic curve in (1) intersects the x-axis. Sketch this curve when $(t_1, x_1) = (2, 1)$.

(c) Suppose that $u(0, y) = \cos(y)$. Find a formula for the solution, $u = u(t, x)$, for $x \geq 0$ and $t \geq 0$.

(4) [10 pts] Consider the PDE for $u = u(t, x)$ given by $u_t + (\sin x)u_x = 0$.
(a) Show that the horizontal lines $x = x(t) = 0$ and $x = x(t) = \pm\pi$ are characteristic curves.

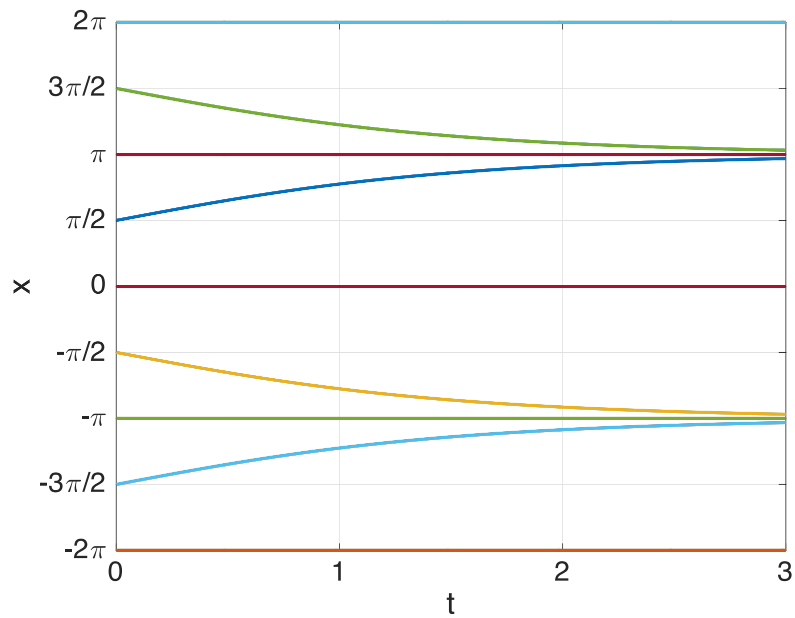
(b) Show that the characteristic, $x = x(t)$, passing through $(t, x) = (0, \pi/2)$ is an increasing function of t.

(c) Suppose now that $u = u(t, x)$ solves the initial value problem

$$u_t + (\sin x)u_x = 0,$$

$$u(0, x) = \begin{cases} 1 & \text{if } |x| \leq \frac{\pi}{2}, \\ 0 & \text{if } |x| > \frac{\pi}{2}. \end{cases}$$

Use the sketch of the characteristic curves below to sketch the solution u at times $t = 1$ and $t = 2$. What is $u_\infty(x) = \lim_{t\to\infty} u(t, x)$?



(5) [10 pts] Solve the initial value problem

$$u_t + 3uu_x = 0, \tag{2}$$

$$u(0,x) = \begin{cases} -2 & \text{if } x < 1, \\ 0 & \text{if } x \geq 1. \end{cases} \tag{3}$$

In particular, identify the subset of the half plane $\{(t,x)|t \geq 0\}$ on which u is determined by (3).

(6) [10 pts] Suppose that $u = u(t,x)$ is a solution of the PDE

$$u_{tt} - c^2 u_{xx} = 0. \tag{4}$$

Let $\xi = x - ct$ and $\eta = x + ct$, and define a function $v = v(\xi, \eta)$ by

$$v(\xi,\eta) = u\left(\frac{\eta - \xi}{2c}, \frac{\eta + \xi}{2}\right). \qquad \text{So, we know that } u(t,x) = v(x - ct, x + ct).$$

Prove that u solves (4) if and only if $v_{\xi\eta} = 0$. Hence show that any solution of (4) is of the form,

$$u(t,x) \;=\; p(x - ct) \;+\; q(x + ct), \qquad \text{for some functions } p \text{ and } q.$$