Math 4362 Homework #5

1. 2.4.1
2. 2.4.4 a,c
3. 2.4.8 a,b
4. 2.4.11
5. [Adapted from 2.4.12] Let $u = u(t, x)$ solve the wave equation, $u_{tt} - c^2 u_{xx} = 0$. Define the momentum density, P, and the energy density, E, of u by

$$P = u_t u_x \qquad \text{and} \qquad E = \frac{1}{2}(u_t^2 + c^2 u_x^2). \tag{1}$$

   (a) Show that $P_t = E_x$ and $E_t = c^2 P_x$.

   (b) Hence show that E and P both satisfy the wave equation.

   (c) Let $\mathbf{P}(t) = \int_{-\infty}^{+\infty} P(t, x)dx$ be the total momentum and $\mathbf{E}(t) = \int_{-\infty}^{+\infty} E(t, x)dx$ be the total energy. Suppose that for each t, as $x \to \pm\infty$, we have that $u_t(t, x) \to 0$ and $u_x(t, x) \to 0$. Show that $\mathbf{P}(t)$ and $\mathbf{E}(t)$ are conserved quantitiies, i.e., that they are constants, independent of time, t.

Recommended Problems [Not to be handed in]

1. 2.4.2
2. 2.4.3
3. 2.4.7
4. 2.4.9