Math 4362 Homework #4

1. Use the method described on pages 8 and 9 of Lecture 3 of the handwritten lecture notes to solve the PDE-IVP

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(xu) = 0, \tag{1}$$

$$u(0, x) = f(x). \tag{2}$$

   In particular, use the method to show that the characteristic curve through the point (t_0, x_0) is given by $x(t) = x_0 e^{t-t_0}$. Sketch some of the characteristic curves in the (t, x)-plane. Then show that the solution to the PDE-IVP is given by $u(t, x) = f(xe^{-t})e^{-t}$. In the case that $f(x) = \frac{1}{1+x^2}$ sketch the solution at times $t = 0$, $t = 1$, and $t = 2$.

2. 2.3.1 (a) and (b)

3. 2.3.2 (a)

4. Consider the PDE-IVP

$$u_t + uu_x = 0, \tag{3}$$

$$u(0, x) = \frac{1}{1 + x^2}. \tag{4}$$

   Write down a formula for the characteristic line that goes through the point y on the x-axis. Sketch some of the characteristic lines in the (t, x)-plane. Calculate the critical time, t_*, at which the solution, u, develops a vertical tangent line. Use the method of characteristics to sketch the solution as a function of x just before and just after the critical time. (Just after the critical time interpret the solution as a multi-valued function.)

5. 2.3.5

6. 2.3.7

Additional Recomended Problems [Do not turn in]

1. 2.3.4

2. 2.3.15. Hint: Show that $u_t + u^2 u_x = -ku\frac{\gamma}{\gamma t+\delta} + ku^{2+\frac{k-1}{k}}\frac{\alpha}{\gamma t+\delta}$.

3. 2.3.16. Specifically, (i) What is the analogue of Equation (2.35) in Olver? (ii) How do Figs. 2.14 and 2.15 in Olver change? (iii) What is the analogue of Equation (2.41) in Olver?