# Math 6301, Fall 2024 Real Analysis

#### Course Information

80327 Math 6301.001 TuTh 2:30-3:45 FN 2.106

#### **Professor Contact Information**

Instructor: John Zweck
Office: FO 3.704J

Email: zweck@utdallas.edu

Webpage: I will maintain a web page for the course linked from the MATH 6301

eLearning course and from http://www.utdallas.edu/~zweck. I will

also communicate with you using a class email list.

Phone: (972) 883-6699 (Do not leave a message. Email me instead.)

Office Hours: Tu 1-2:15 and by appointment. If you cannot come to my office hours

please contact me in class or by email to set up a time to meet. Also,

you are encouraged to ask me questions by email.

# Course Pre-requisites

MATH 5302

# Course Description

Lebesgue measure in finite-dimensional spaces, abstract measures, measurable functions, convergence a.e., Egorov's Theorem, convergence in measure, Lebesgue integral, Lebesgue's bounded convergence theorem, Levi's monotone convergence theorem, Fatou's Lemma, Fubini's theorem,  $L^p$ -spaces.

# **Student Learning Outcomes**

- 1. State the definitions of fundamental concepts in Lebesgue theory and apply them in proofs
- 2. State and apply fundamental theorems on the existence and properties of Lebesgue measure and integral
- 3. Calculate concrete integrals, rigorously justifying each step using the theory

- 4. Apply Lebesgue integration theory to prove results about specific classes of functions
- 5. Construct examples that illustrate aspects of the theory
- 6. Reproduce proofs of major results in the theory
- 7. Construct proofs of known results that expand upon the theory discussed in lectures
- 8. Compare the Lebesgue and Riemann theories of integration

#### **Textbooks**

The following texts are recommended and represent a range of perspectives and levels of sophistication. The lectures will mostly be based on [J] and [A].

- [A] "Measure, Integration and Real Analysis", S. Axler, Springer Graduate Texts in Mathematics, 2023.
- [J] "Lebesgue Integration on Euclidean Space", F. Jones, Jones and Bartlett, 1993
- [K] Introduction to Mathematical Analysis, W. Krawcewicz et al.
- [WZ] "Measure and Integral: An Introduction to Real Analysis", R.L. Wheeden and A. Zygmund, Marcel Dekker, Inc. 1977

## **Grading Policy**

Grades: Homework 40%, Midterm Exam I 15%, Midterm Exam II 15% Final

Exam 30%

Homework: Assigned most weeks. Must be handed in at start of class on due

date.

Midterm Exam I: Thursday Oct 3 (8:30am) to Saturday Oct 5 (12 noon) in Testing

Center [120 minutes]

Midterm Exam II: Thursday Nov 7 (8:30am) to Saturday Nov 9 (12 noon) in Test-

ing Center [120 minutes]

Final Exam: Tuesday Dec 10th from 2:00pm-4:45pm in FN 2.106. The final exam

will be based on the whole course.

Sign up for a seat in testing center for two midterm exams ASAP starting Monday Aug 19th!

# Academic Calendar and Assignments

The Lecture Notes and Homework Assignments will be posted on the course web page. Most of the problems will be graded.

# Daily Schedule [subject to change!]

Day	Topic	Book & Section
Tu Aug 20	Compact sets	<b>J</b> : 1.C-D
Th Aug 22	Continuous functions	<b>J</b> : 1.E
Tu Aug 27	Course overview	
Th Aug 29	Riemann Integral	<b>A</b> : 1.A
Tu Sep 3	Why do we need Lebesgue?	<b>A</b> : 1.B, <b>J</b> : 4.A
Th Sep 5	Construction of Lebesgue Measure: Polygons	<b>J</b> : 2.A
Tu Sep 10	Construction of Lebesgue Measure: Open Sets	<b>J</b> : 2.A
Th Sep 12	Construction of Lebesgue Measure: Open Sets	<b>J</b> : 2.A
Tu Sep 17	Construction of Lebesgue Measure: Compact Sets	<b>J</b> : 2.A
Th Sep 19	The Cantor Set	<b>A</b> : 2.D, <b>J</b> : 2.A
Tu Sep 24	Inner & Outer Measure	<b>J</b> : 2.A
Th Sep 26	Lebesgue Measure and It's Properties	<b>J</b> : 2.A, 2.B
	Existence of Nonmeasurable Set	
Tu Oct 1	$\sigma$ -Algebras, Borel Sets	<b>J</b> : 5.A, 5.B, 5.C
Th Oct 3	Measurable Functions	<b>J</b> : 5.D
Tu Oct 8	Simple Functions	<b>J</b> : 5.E
	Catch Up	
Th Oct 10	Convergence a.e.	
	Convergence in Measure	<b>WZ</b> : 4.4
Tu Oct 15	Egorov's Theorem	<b>A</b> : 2.E
Th Oct 17	Luzin's Theorem	<b>A</b> : 2.E
Tu Oct 22	Lebesgue Integral: Nonnegative Functions	<b>J</b> : 6.A
	Monotone Convergence Theorem	
Th Oct 24	Fatou's Lemma	<b>J</b> : 6.A
Tu Oct 29	Lebesgue Integral: General Measurable Functions	<b>J</b> : 6.B
Th Oct 31	Lebesgue Dominated Convergence Theorem	<b>J</b> : 6.B
Tu Nov 5	Almost Everywhere	<b>J</b> : 6.C
Th Nov 7	Examples	<b>J</b> : 6.D, 6.G
	Abstract Measure Spaces	<b>J</b> : 6.E, 6.F
Tu Nov 12	Fubini's Theorem	<b>J</b> : 8
	Riemann implies Lebesgue	<b>A</b> : 3.B
Th Nov 14	$L^p$ Spaces	<b>A</b> : 7.A
Tu Nov 19	$L^p$ Spaces	<b>A</b> : 7.A
Th Nov 21	$L^p$ is a Banach Space	<b>A</b> : 7.B
Tu Dec 3	Differentiation, Hardy-Littlewood Maximal Function [if time]	<b>A</b> : 4.A
Th Dec 5	Lebesgue Differentiation Theorem, FTC [if time]	<b>A</b> : 4.B

### Instructor Policies

#### Homework

Homework must be submitted on paper at the start of class each day it is due. Please layout your work so as to leave sufficient room for the grader to add written comments. Students who turn in a homework set late (after 2:30pm on the due date) will be penalized 10% unless they request an extension prior to 2:30pm. You may ask me questions about the homework and you may discuss a first draft of your solutions with another student in the class. However the final version must be your own. Students may be granted a second submission on selected homework questions.

#### Making up an exam you missed

If you miss one of the exams you *may* be given the chance to take a make up exam. To request a make up you should contact me **no later than 48 hours after** the exam time. Generally speaking, you will be offered a make up if you are sick or if a close relative or friend is gravely injured/sick or dies. However I will listen to all reasonable requests. Be prepared to bring appropriate evidence in support of your request.

## **Academic Integrity**

I will be vigorous in reporting all instances of cheating to the University administration. See http://www.utdallas.edu/deanofstudents/dishonesty/

# UT Dallas Syllabus Policies and Procedures

The information at http://go.utdallas.edu/syllabus-policies constitutes the University's policy abd procedures segement of the course syllabus.

The descriptions and timelines contained in this syllabus are subject to change at the discretion of the Professor.