Math 6301 Homework 7

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1. **A**.2E: 5

2. Suppose that $\|f_n - f\|_\infty \to 0$ as $n \to \infty$. Show that $f_n \to f$ pointwise almost everywhere. **Hint:** Mimic the proof that L^∞ is complete (Theorem 15).

3. Give an example to show that pointwise almost everywhere convergence does not imply L^∞-convergence.

4. Let $X \subset \mathbb{R}^n$ be measurable and let $f_n, f : X \to \mathbb{R}$ be measurable. Show that if f_n is Cauchy in measure and there is a subsequence so that $f_{n_k} \stackrel{m}{\to} f$, then $f_n \stackrel{m}{\to} f$.

5. Let $X \subset \mathbb{R}^n$ be measurable and let $f_n, f, g_n, g : X \to \mathbb{R}$ be measurable. Prove that

   (a) If $f_n \stackrel{m}{\to} f$ and $f_n \stackrel{m}{\to} g$ then $f = g$ almost everywhere.

   (b) If $f_n \stackrel{m}{\to} f$ and $g_n \stackrel{m}{\to} g$ then $f_n + g_n \stackrel{m}{\to} f + g$.

   (c) **Extra Credit:** If $\lambda(X) < \infty$ and $f_n \stackrel{m}{\to} f$ and $g_n \stackrel{m}{\to} g$ then $f_n g_n \stackrel{m}{\to} fg$.
   Hints:
   (i) $f_n g_n - fg = (f_n - f)(g_n - g) + f(g_n - g) + g(f_n - f)$.
   (ii) Suppose $|h_1(x)h_2(x)| > \epsilon$. Then there is a rational r so that $|h_1(x)| > r > \frac{\epsilon}{|h_2(x)|}$.

Additional Problem [Not not hand in]

1. **A**.2E: 1

2. **A**.2E: 3

3. Give an example to show why we need $\lambda(X) < \infty$ in 5(c) above.