Math 6301 Some Extra Problems

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1. Fill in all the ”you can check” type gaps in the lecture notes.
2. **J**.2B: 28
3. **J**.2B: 32
4. **J**.2B: 33
5. **J**.2B: 35
6. **J**.2B: 37
7. **J**.2B: 38
8. **J**.5: 1
9. **J**.5: 5
10. **J**.5: 13
11. **J**.5: 14
12. **J**.5: 16abc
13. **A**.2A: 12
14. **A**.2B: 1
15. **A**.2B: 3
16. **A**.2B: 7
17. **A**.2B: 8
18. **A**.2B: 12
19. **A**.2B: 14
20. **A**.2B: 18
21. **A**.2B: 28 ,
22. **A**.2D: 5
23. **A**.2D: 7
24. **A**.2E: 1
25. **A**.2E: 9
26. Prove $[a, b)$ is Borel
27. Prove every countable set is Borel

28. Let f be a simple function taking its distinct values on disjoint sets $E_1, \cdots, E_n$. Show that f is measurable if and only if $E_1, \cdots, E_n$ are measurable.

29. Let A and B be bounded subsets of $\mathbb{R}$ such that

$$\inf\{|x-y| : x \in A, y \in B\} > 0.$$

Show that $\lambda^*(A \cup B) = \lambda^*(A) + \lambda^*(B)$.

30. Let G be an open set so that $A \subseteq G$ and $G \cap B = \emptyset$. Show that $\lambda^*(A \cup B) = \lambda^*(A) + \lambda^*(B)$.

31. Use definition of a measurable function to show that any constant function is measurable.

32. Let S be a dense subset of $\mathbb{R}$. Show that f is measurable iff $\{x : f(x) > c\}$ is measurable for all $c \in S$.

33. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. Show that f' is measurable. Hint: Use limit definition of derivative and fact that the translate of a measurable set is measurable.

34. Let

$$f_n(x) = \begin{cases} 1 & \text{if } \frac{1}{n} \le |x| \le \frac{2}{n} \\ 0 & \text{otherwise.} \end{cases}$$

Show that $f_n \to 0$ on $[-2, 2]$ but that there is no set, S, of measure zero so that $f_n \to 0$ uniformly on $[-2, 2] \sim S$. For all $\epsilon > 0$ explicitly construct a measurable set B for which conclusions of Egorov's theorem hold.

35. Suppose $f_n \to f$ pointwise on $[0, 1]$ and that each f_n is measurable. Use Egorov's Theorem to show that there is a sequence of sets A_n in $[0, 1]$ so that $\lambda([0, 1] \sim \cup_{n=1}^{\infty} A_n) = 0$ and $f_n \to f$ uniformly on eachh A_n.