LAST NAME:		FIRST NAME:		

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MATH 4355 [Spring 2020] Exam II, Apr 10-12 Take Home

This is an open book, open notes exam. You may use material from our course, including on the course web page. You may not consult any outside sources (for example on other web pages); in any case the exam is designed so that you will not find answers on the web. **Except for the course instructor, you may not discuss the contents of this exam with any other person in any way until your exam grade has been posted in eLearning. No use of calculators or mathematical software is allowed. Show all hand calculations. *Give complete explanations!!* The exam must be completed and uploaded in eLearning by Sunday April 12th at 5pm (Dallas time).** Although the exam is designed so that most students should be able to complete it in about 90 minutes, there is no time limit, apart from the submission deadline above. Call me at (972) 883-6699 or email `zweck@utdallas.edu` if you have any questions.

Please sign the pledge:

> I pledge that I have not discussed the contents of this exam with any other person in any way and will not do so until my exam grade is posted in eLearning.

Signature: ________________________________

Please write your answers on separate sheets of paper and if you have a printer include this cover page (filled out) in your scanned solutions. If you do not have a printer, please handwrite the relevant parts above on the first page of your submission.

(1) [15 pts] Let $\mathbf{T} : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation and let $\mathcal{B}$ and $\mathcal{B}'$ be two bases for $\mathbb{R}^3$.

(a) Explain whether or not it is possible for

$$[\mathbf{T}]_{\mathcal{B}} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \qquad \text{and} \qquad [\mathbf{T}]_{\mathcal{B}'} = \begin{pmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{pmatrix}.$$

(b) Explain whether or not it is possible for

$$[\mathbf{T}]_{\mathcal{B}} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \qquad \text{and} \qquad [\mathbf{T}]_{\mathcal{B}'} = \begin{pmatrix} 9 & 7 & 8 \\ 3 & 1 & 2 \\ 6 & 4 & 5 \end{pmatrix}.$$

(c) Suppose that $\mathbb{R}^3$ is equipped with the standard inner product and that

$$[\mathbf{T}_{\mathcal{B}}] = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

State any conditions that must be placed on the basis $\mathcal{B}$ to guarantee that

$$[\mathbf{T}^*]_{\mathcal{B}} = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}.$$

(2) [20 pts] Let

$$\mathbf{A} = \frac{1}{3}\begin{pmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{pmatrix}, \qquad \mathbf{B} = \frac{1}{3}\begin{pmatrix} 2 & 2 & -1 \\ 0 & 3 & 0 \\ -2 & 4 & 1 \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Provide careful explanations for which of these three matrices are and which are not

(a) a change of basis matrix,

(b) the matrix of a reflection,

(c) the matrix of a projector,

(d) orthogonal.

(3) [10 pts] Let $\mathcal{X}$ be the vector subspace of $\mathbb{R}^3$ defined by the plane $x_1 + 2x_2 + 3x_3 = 0$ and let

$$\mathcal{Y} = \text{Span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}.$$

Calculate the projector onto $\mathcal{X}$ along $\mathcal{Y}$.

(4) [15 pts] Let $\mathbb{C}^{n\times n}$ be the vector space of $n \times n$ matrices with complex entries. Let $\mathbf{Tr} : \mathbb{C}^{n\times n} \to \mathbb{C}$ be defined by $\mathbf{Tr}(\mathbf{A}) = \sum_{k=1}^{n} A_{kk}$. (Here A_{kl} denotes the (k,l)-entry of the matrix $\mathbf{A}$.)

(a) Prove that $\langle \mathbf{A} | \mathbf{B} \rangle = \mathbf{Tr}(\mathbf{A}^*\mathbf{B})$ is an inner product on $\mathbb{C}^{n\times n}$.

(b) Let $\{\mathbf{e}_1, \cdots, \mathbf{e}_n\}$ be the standard basis for $\mathbb{C}^n$. Let $\mathbf{E}_{ij}$ be the $n \times n$ matrix defined by $\mathbf{E}_{ij} = \mathbf{e}_i\mathbf{e}_j^*$. Prove that

$$\{\mathbf{E}_{ij} \,|\, i,j = 1, \cdots, n\}$$

is an orthonormal basis for $\mathbb{C}^{n\times n}$.

(c) Let $\mathbf{Tr}^*$ denote the adjoint of the linear transformation $\mathbf{Tr}$. Calculate $\mathbf{Tr}^*(1)$. Carefully justify each step in your calculation.