NAME:

1 /15	2 /6	3 /16	4 /10	5 /10	6 /8	7 /10	T /75

MATH 430 (Fall 2008) Exam 1, Sep 29

No calculators, books or notes! Show all work and give **complete explanations**.
This 75 minute exam is worth a total of 75 points.

(1) [15 pts]
(a) Define the nullspace and range of a matrix.

(b) State the Rank and Nullity Theorem, and illustrate what it says in the context of a well-chosen example.

(c) Define the concept of a maximal linearly independent subset of a finite dimensional vector space.

(2) [6 pts] Let $\mathbf{A}$ be a block matrix of the form $\mathbf{A} = \begin{pmatrix} \mathbf{B} \\ \mathbf{C} \end{pmatrix}$. Prove that $N(\mathbf{A}) = N(\mathbf{B}) \cap N(\mathbf{C})$.

(3) [16 pts] Let $\mathbf{A}$ be the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 9 & 7 \\ 3 & 7 & 4 \\ 8 & 18 & 14 \end{pmatrix}.$$

Find bases for the four fundamental subspaces of $\mathbf{A}$.

(4) [10 pts]

(a) Prove that if a matrix is both symmetric and skew-symmetric then it is zero.

(b) *Without using matrices* prove that the composition of two linear mappings between vector spaces is linear.

(5) [10 pts]
(a) Let $\mathbf{A}$ be $m \times n$ and $\mathbf{B}$ be $n \times \ell$. Prove that each column of $\mathbf{AB}$ can be expressed as a linear combination of the columns of $\mathbf{A}$. In particular, find the coefficients in these linear combinations.

(b) Let

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{pmatrix}$$

Use the formula you derived in (a) to calculate the 3rd column of $\mathbf{AB}$.

(6) [8 pts] For each of the following statements either prove that the statement is true or give a *specific* counterexample.

(a) The union of two vector subspaces of a vector space is a vector subspace.

(b) The intersection of two vector subspaces of a vector space is a vector subspace.

(7) [10 pts] Find a basis for the vector space consisting of all 3×3 skew-symmetric matrices and prove that it is indeed a basis.

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________