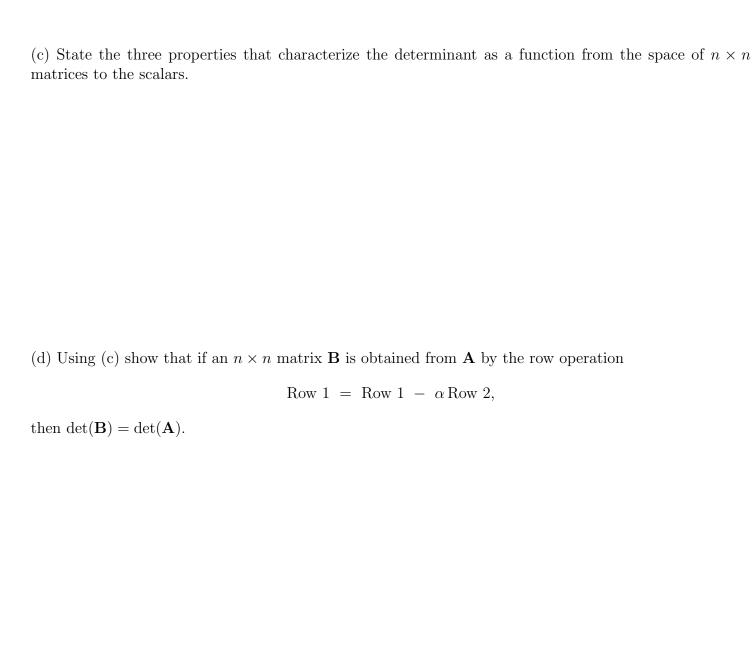
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MATH 430 (Fall 2006) Exam II, November 1st

Show all work and give **complete explanations** for all your answers. This is a 75 minute exam. It is worth a total of 100 points.

- (1) [32 pts]
- (a) Prove that similar matrices have the same spectrum.

(v	(b) Use the result of (a) to well defined. (Here \mathcal{V} is a	o define the spectrum finite dimensional vec	of a linear transforetor space.)	mation $\mathbf{T}: \mathcal{V} \to \mathcal{V}$ a	nd prove that it is



$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right\}$$

and

$$\mathcal{B}' = \left\{ \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$$

be two bases for $\mathcal{V} = \mathcal{R}^2$. Calculate the matrix $[\mathbf{T}]_{\mathcal{BB}}$ of the change of basis linear transformation $\mathbf{T}: \mathcal{V} \to \mathcal{V}$ from \mathcal{B} to \mathcal{B}' .

(b) Suppose that
$$\mathbf{v} \in \mathcal{V}$$
 has coordinate vector $[\mathbf{v}]_{\mathcal{B}} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$. What is $[\mathbf{v}]_{\mathcal{B}'}$?

(3) [10 pts] Let \mathbf{A} be the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 4 & 5 & 8 \end{pmatrix}.$$

Calculate $det(\mathbf{A})$ using

(a) Row operations

(b) A cofactor expansion

(4) [16 pts] Use eigenvalues and eigenvectors to solve the initial value problem

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x}$$
$$\mathbf{x}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix},$$

where
$$\mathbf{A} = \begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix}$$
.

- (5) [16 pts]
- (a) Use the formula for the determinant of a block matrix to prove that if **B** is $m \times n$ and **C** is $n \times m$ then

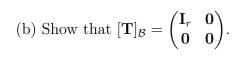
$$\lambda^m \det(\lambda \mathbf{I}_n - \mathbf{C}\mathbf{B}) = \lambda^n \det(\lambda \mathbf{I}_m - \mathbf{B}\mathbf{C})$$

for all scalars λ .

(b) Use the result of (a) to show that if $n = m$ then BC and CB have the same spectrum, $\sigma(\mathbf{BC}) = \sigma(\mathbf{CB})$.

(c) Construct a counterexample to show that $\sigma(\mathbf{BC}) \neq \sigma(\mathbf{CB})$ when $n \neq m$.

- (6) [16 pts] Suppose that $\mathbf{T}: \mathcal{V} \to \mathcal{V}$ is a linear transformation such that $\mathbf{T}^2 = \mathbf{T}$. Let $\{\mathbf{x}_1, \dots, \mathbf{x}_r\}$ be a basis for $\mathcal{R}(\mathbf{T})$ and $\{\mathbf{y}_1, \dots, \mathbf{y}_{n-r}\}$ be a basis for $\mathcal{N}(\mathbf{T})$, where $n = \dim \mathcal{V}$.
- (a) Show that $\{\mathbf{x}_1, \cdots, \mathbf{x}_r, \mathbf{y}_1, \cdots, \mathbf{y}_{n-r}\}$ are linearly independent and hence form a basis \mathcal{B} for \mathcal{V} . [Hint: Show $\mathbf{T}\mathbf{x} = \mathbf{x}$ for all $\mathbf{x} \in \mathcal{R}(\mathbf{T})$.]



(c) Use (1b) to calculate the spectrum of the linear transformation T.

 ${\bf Pledge:}\ I\ have\ neither\ given\ nor\ received\ aid\ on\ this\ exam$

Signature: