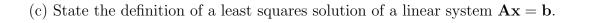
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			N	IATH 4	430 (Fall 2	2006) I	Exar	n 1, Oc	tobe	er 4th				
Show a	all work	and s	give con	nplete e	exnla	natio	ns for a	ll voi	ır answei	S					

Show all work and give **complete explanations** for all your answers.

This is a 75 minute exam. It is worth a total of 100 points.

- (1) [30 pts]
- (a) Define the term $\it maximal\ linearly\ independent\ set.$

(b) State the Basis Characterization Theorem.



(d) Suppose that $\mathbf{B}_{r \times r}$ is an invertible $r \times r$ matrix and that $\mathbf{0}_{p \times q}$ is the $p \times q$ zero matrix. Let \mathbf{A} be the square matrix

$$\mathbf{A} \; = \; egin{pmatrix} \mathbf{B}_{r imes r} & \mathbf{0}_{r imes s} \ \mathbf{0}_{s imes r} & \mathbf{0}_{s imes s} \end{pmatrix}.$$

Find bases for the nullspace, $N(\mathbf{A})$, and the range, $R(\mathbf{A})$, of \mathbf{A} and verify that the Rank and Nullity Theorem holds for \mathbf{A} .

((e) Suppose that $N(A)$	$\mathbf{A}) = N(\mathbf{B})$) for two matrices	A and B.	Must it be tr	ue that Rank(A	(= Rank(\mathbf{B})	?

(f) Let $\{\mathbf{v}_1, \cdots, \mathbf{v}_n\}$ be a basis for a vector space \mathcal{V} and let $\mathbf{u} \in \mathcal{V}$. Prove that there are unique scalars $\alpha_1, \cdots, \alpha_n \in \mathcal{R}$ so that

$$\mathbf{u} = \alpha_1 \mathbf{v}_1 + \dots + \alpha_1 \mathbf{v}_n.$$

(2) [15 pts] Let \mathbf{A} be the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 4 & 1 & 3 \\ 3 & 6 & 1 & 4 \end{pmatrix}.$$

Find bases for the null space and range of the $\bf A$ and for the range of $\bf A^T.$ (3) [15 pts] Find the least squares solutions to the linear system

$$2x + 3y = 2$$

$$4x - 2y = -1$$

$$x + 5y = 1$$

$$2x + 0y = 3$$

(4) [10 pts] Let **A** and **B** be two matrices with the same number of columns and let $\mathbf{C} = \begin{pmatrix} \mathbf{A} \\ \mathbf{B} \end{pmatrix}$. Prove that $N(\mathbf{C}) \subseteq N(\mathbf{A})$. Is $N(\mathbf{C}) = N(\mathbf{A})$? Why?

(5) [10 pts] Suppose that ${\bf A}$ is a 2×2 matrix with

$$\mathbf{A} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \qquad \mathbf{A} \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}.$$

Without working out the entries of **A**, find $\mathbf{A} \begin{pmatrix} 6 \\ 10 \end{pmatrix}$.

(6) [10 pts] Let **A** be an $n \times n$ matrix. Suppose that $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ is a basis for \mathbb{R}^n such that $\{\mathbf{v}_{r+1}, \dots, \mathbf{v}_n\}$ is a basis for $N(\mathbf{A})$. Prove that $\{\mathbf{A}\mathbf{v}_1, \dots, \mathbf{A}\mathbf{v}_r\}$ is a basis for $R(\mathbf{A})$.

(7) [10 pts] Let \mathbf{v} , \mathbf{w} be two column vectors in \mathbb{R}^n a that $\mathbf{w}^T\mathbf{v} \neq 1$. Show that the matrix $I - \mathbf{v}\mathbf{w}^T$ is in $I - c\mathbf{v}\mathbf{w}^T$, for some scalar c . Also, find a formula for	vertible and that its inverse is a matrix of the form
Pledge: I have neither given nor received aid on this	exam
Signature:	