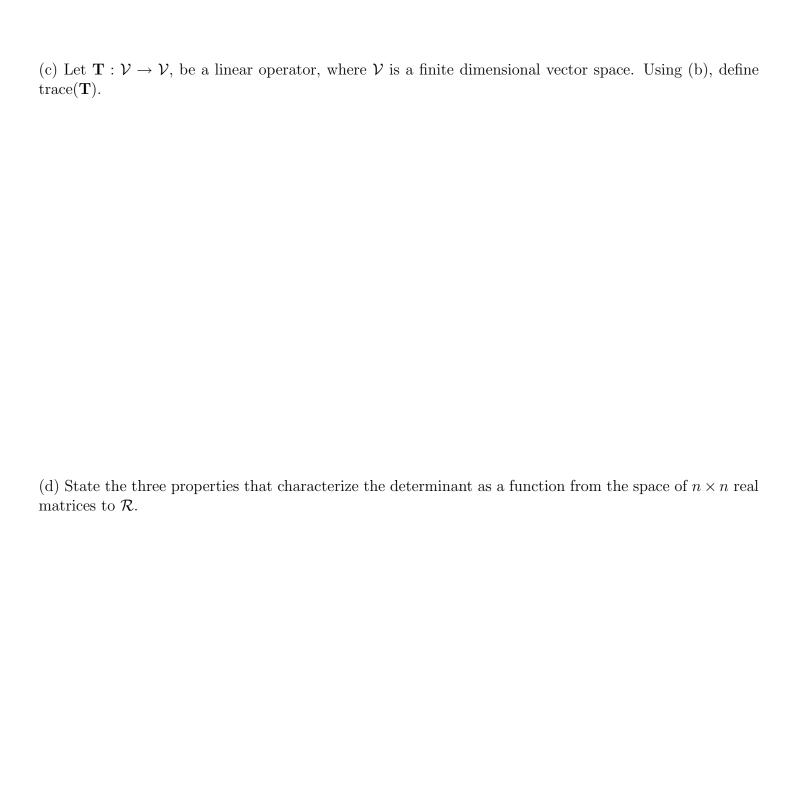
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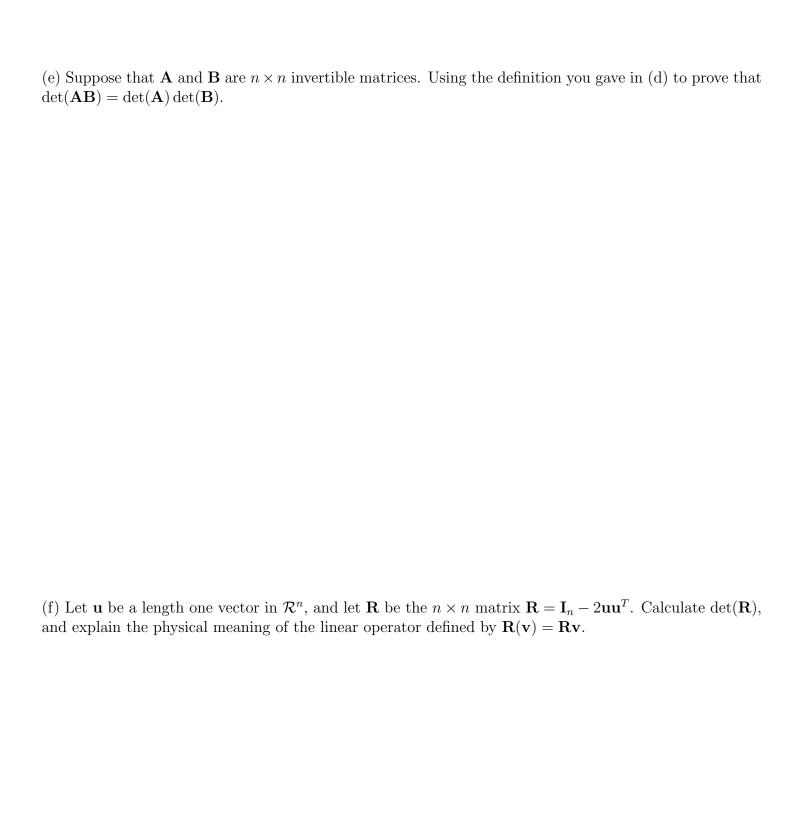
## MATH 430 (Fall 2005) Exam 2, November 3rd

Show all work and give **complete explanations** for all your answers. This is a 75 minute exam. It is worth a total of 100 points.

- (1) [30 pts]
- (a) Let **u** be a non-zero  $n \times 1$  column vector and **v** a non-zero  $m \times 1$  column vector. Prove that  $\mathbf{u}\mathbf{v}^T$  has rank 1.

(b) Suppose that **A** and **B** are  $n \times n$  matrices. Prove that trace(**AB**) = trace(**BA**).





- (2) [10 pts] True or false? If true give a brief justification. If false provide a counterexample.
- (a)  $\det(\mathbf{A} + \mathbf{B}) \det(\mathbf{A} \mathbf{B}) = \det(\mathbf{A}^2 \mathbf{B}^2)$ .

(b) Let  $\mathbf{v} = (2,3)^T$ . In the standard basis  $\mathcal{B}$  for  $\mathcal{R}^2$ , the matrix of the projection operator  $\mathbf{P}_{\mathbf{v}} : \mathcal{R}^2 \to \mathcal{R}^2$  onto the span of  $\mathbf{v}$  is

 $[\mathbf{P}_{\mathbf{v}}]_{\mathcal{B}} = \begin{pmatrix} 4 & 6 \\ 6 & 9 \end{pmatrix}.$ 

(3) [12 pts] For the linear operator  $\mathbf{T}: \mathcal{R}^2 \to \mathcal{R}^2$  defined by  $\mathbf{T}(x,y) = (x-y,2x+4y)$ , calculate the matrix,  $[\mathbf{T}]_{\mathcal{B}}$ , of  $\mathbf{T}$  in the basis  $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$ .

(4) [18 pts] Let  $\mathbf{P}$  be the matrix

$$\mathbf{P} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 6 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

- (a) Calculate  $\det(\mathbf{P})$  using
- (i) Row operations

(ii) Block determinants based on the blocking

$$\mathbf{P} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}, \quad \text{where } \mathbf{A} \text{ is } 1 \times 1 \text{ and } \mathbf{D} \text{ is } 2 \times 2.$$

(iii) A cofactor expansion.

(b) What is  $\det(\mathbf{P}^T\mathbf{P})$ , and why?

(5) [10 pts] Let  $\mathbf{T}: \mathcal{V} \to \mathcal{W}$  be a linear transformation between finite-dimensional vector spaces  $\mathcal{V}$  and  $\mathcal{W}$ . Let  $\mathcal{B}$  be a basis for  $\mathcal{V}$  and let  $\mathcal{B}'$  be a basis for  $\mathcal{W}$ . Define the matrix  $[\mathbf{T}]_{\mathcal{BB}'}$  of  $\mathbf{T}$  with respect to these two bases, and prove that

$$[\mathbf{T}(\mathbf{u})]_{\mathcal{B}'} \ = \ [\mathbf{T}]_{\mathcal{B}\mathcal{B}'}[\mathbf{u}]_{\mathcal{B}}.$$

(6) [8 pts] Suppose <b>A</b> is a square matrix whose entries are differentiable functions of a real variable $t$ , that is, $\mathbf{A}_{ij} = \mathbf{A}_{ij}(t)$ . Prove that det <b>A</b> is also a differentiable function of $t$ .									

(7) [12 pts] The least squares quadratic fit to m data points  $(t_1, y_1), (t_2, y_2), \dots (t_m, y_m)$  in  $\mathcal{R}^2$  is the quadratic function  $y = f(t) = \alpha + \beta t + \gamma t^2$  for which the parameter vector  $(\alpha, \beta, \gamma)$  is the global minimum of the function

$$Q = Q(\alpha, \beta, \gamma) = \sum_{i=1}^{m} (\alpha + \beta t_i + \gamma t_i^2 - y_i)^2.$$

(a) Let  $\mathbf{x} = (\alpha, \beta, \gamma)^T$ . Find an  $m \times 1$  vector  $\mathbf{y}$  and an  $m \times 3$  matrix  $\mathbf{A}$  so that

$$Q = Q(\mathbf{x}) = \|\mathbf{A}\mathbf{x} - \mathbf{y}\|^2.$$

(b) By differentiating $Q(\mathbf{x})$ with respect to the <i>i</i> -t satisfies the normal equations $\mathbf{A}^T \mathbf{A} \mathbf{x} = \mathbf{A}^T \mathbf{y}$ .	sh coordinate $\mathbf{x}_i$ of $\mathbf{x}$ , prove that the minimizer of $Q$
Pledge: I have neither given nor received aid on this	$s \ exam$
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