A Menagerie of Mathematical Models

Active Learning Project #1
Circular Paraboloids

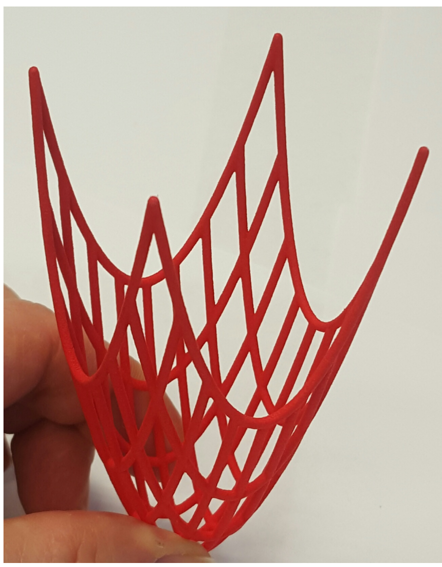
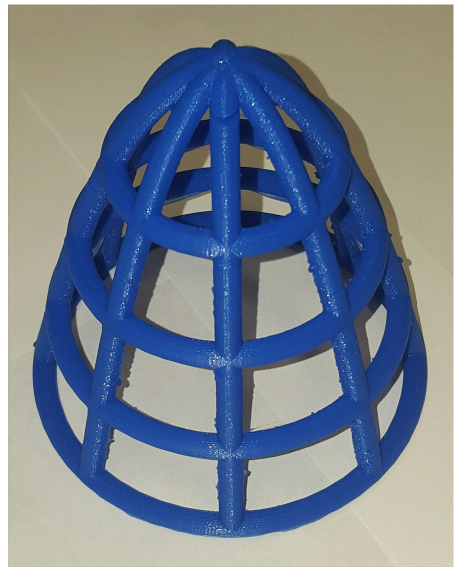

Figure 1: Left: **Red** Circular Paraboloid; Right: **Blue** Circular Paraboloid. For this worksheet your group will need one blue model and one red model. **Rules:** *We want these models to last for many generations of students. Since some of them are delicate please handle with care. Try not to transfer any white board marker ink on your hands to the models. Do not attempt to flex the models. When you have completed the activity put the model back in the box you got it from.*

The purpose of this activity is to investigate the geometric structure of the surface given by the graph of the function

$$z = f(x, y) = x^2 + y^2.$$

The mathematical name for this surface is a **circular paraboloid**. The common name is a **bowl**. In reality the surface is an infinitely thin sheet. However, we represent the surface using a **mesh**. You can think of a **surface mesh** as being like a curved fishing net that is draped over the surface. You could imagine making such a fishing next by laying a collection of ropes on the surface, some going one way on the surface, some going another way. Wherever two ropes cross you tie them together. With the red and blue 3D models we have used thin plastic tubes instead of ropes, but the idea is the same.

1. For each model identify where the origin is and which direction the three coordinate axes go. You may find it helpful to draw x and y coordinate axes on a sheet of paper, put the paper on the table, and position the model on top of the paper. (The z-axis would of course be perpendicular to the paper.)

2. The domain of the function $f(x, y) = x^2 + y^2$ is the entire xy-plane. In other words, for **every** point, (x, y), in the horizontal plane there is a point immediately above it on the surface. For

example, what is the z-coordinate of the point on the surface that is immediately above the point $(x, y) = (2, 3)$ in the plane?

3. If we made a model of the entire surface it would be an infinite sheet. Since we can't do that in practice we need to **restrict the domain** of the function, f, to be a subset, D, of the xy-plane, and just make a model of that portion of the surface that lies above this set D.

   What is the domain, D, for the red model? What about for the blue model?

   To answer these questions sit each model on the table and use the flashlight app on your phone to cast a shadow straight down onto the table. *Whenever you use a flashlight to cast a shadow in these projects you need to have the* **flashlight at least two feet above the model** *so that the rays of light are falling almost vertically down onto the model.*

4. Carefully make sketches showing the shadow of the two meshed surfaces on the table.

5. The red model is made using **two families of curves**. Explain how each of these curves can be obtaining by slicing the surface in a plane. What are the equations for these curves?

   Hint: The sketch you drew in step 4 may be helpful here.

   Textbooks often call these curves ”traces”, but we will use the more descriptive term ”slices”.

6. By looking at the red model, explain how these curves can all be obtained by taking a single curve and moving it around in space. Use the equations for the curves you obtained in the previous step to come to the same conclusion.

7. Next let's turn our attention to the blue model. Explain how each of the curve in the blue model can be obtaining by slicing the surface in a plane. What are the equations for these curves?

8. What symmetry property of the circular paraboloid is shown more clearly with the blue model than with the red model?

9. The circles in the blue model are equally spaced in the vertical direction but as you can see from the sketch you drew in step 4 their shadows on the horizontal plane are not equally spaced. Why?

10. **Geometric Imagination (GI) Builder:** [*Do this after class*] Try to visualize and sketch a **different surface** similar to blue circular paraboloid that is built out of circles that are equally spaced in the vertical direction and whose shadows on the horizontal plane are also **equally** spaced.