# A Menagerie of Mathematical Models

## Active Learning Project #3
## Helices

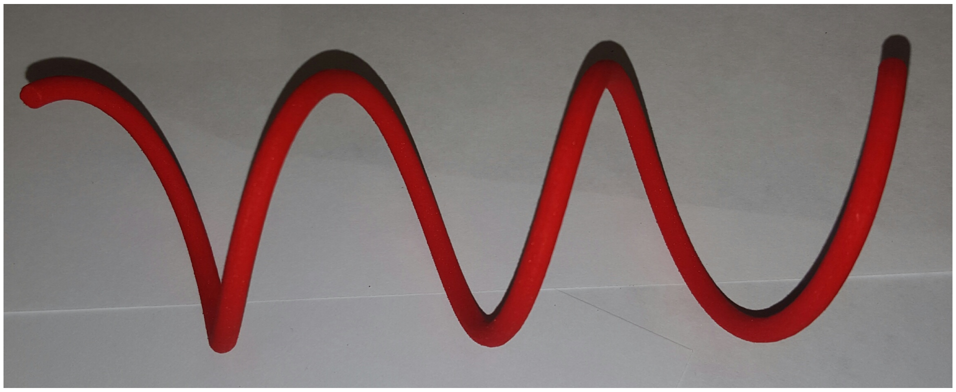

Figure 1: The **helix curve**. For this worksheet your group will need a zip-lock bag containing three red helices. **Rules:** *We want these models to last for many generations of students. Since some of them are delicate please handle with care. Try not to transfer any white board marker ink on your hands to the models. Do not attempt to flex the models.* **At the end of the project put these three helices back together in the same bag.**

**Assessment:** If this project is being assessed, your small group needs to show the Teaching Assistant (TA) your answers to the questions labeled **TACheck**. Total Points: 10.

The purpose of this activity is to investigate the geometric structure of the curve whose parametrization is given by

$$(x, y, z) = \mathbf{r}_1(t) = (\cos t, \sin t, t). \tag{1}$$

The name for this curve is a **helix**. Springs and corkscrews are a helical curves and a DNA molecule is a double helix.

1. **TACheck [1 pt]: Explanation.** Two of the helices in your zip-lock bag are identical to each other, but the third is different. Find the identical pair. In what way is the third helix different from the first two? These two types of helices are called **right-handed** and **left-handed** helixes.

2. **TACheck [1pt]: Explanation.** The parametrized curve given by equation (1) is a right-handed helix. Use this fact to identify which models are left-handed and which are right-handed helices. **Hint:** Imagine you are an ant walking along the curve. Examine which way the curve rotates as you walk up it.

3. **TACheck [1pt]: Algebra.** By wrapping one of the curves in a rolled up sheet of paper convince yourself that the helix is a curve that lies on a cylinder. Do an algebraic calculation to come to the same conclusion.

4. **TACheck [1pt]: Value of $b$.** If we let the parameter, $t$, vary over the entire real number line then we would get an infinitely long helix. However to make the model we need to **restrict**

the values of $t$ to lie in a closed interval, $a \leq t \leq b$. When we do this we get a curve that should really be called a **segment of a helix**. We can choose the origin of our coordinate system so that $a = 0$. If we do that, what is the value of $b$? **Hint:** How many complete turns does our model of the helix make?

5. **TACheck [1 pt]: Show.** The left-handed helix is a mirror image (reflection) of the right-handed helix. Show that there are (at least) two distinct ways to position the helices so that one is a mirror image of the other.

6. In this question we are going to forget about helices and make sure we understand reflections. When we talk about a reflection in a plane you should think of a reflection created by a mirror that lies in the given plane. What is the reflection of the point $(x, y, z)$ in the plane $z = 0$? What is the reflection of the point $(x, y, z)$ in the plane $x = 0$?

7. **TACheck [2pts]: Parametrizations.** Use Equation (1) together with your answers to 5 and 6 to work out two different parametrizations for the left-handed helix.

8. **TACheck [3pts]: Show and explain.** The parametrization

$$(x, y, z) \;=\; \mathbf{r}_2(t) \;=\; \mathbf{r}_1(t + \tfrac{\pi}{2}) \;=\; (\cos(t + \tfrac{\pi}{2}), \sin(t + \tfrac{\pi}{2}), t + \tfrac{\pi}{2}), \tag{2}$$

   is a parametrization of another segment of a helix. Use the two right-handed helix models to show how to position the helix segment with parametrization $\mathbf{r}_2$ relative to the helix segment with parametrization $\mathbf{r}_1$. [**Hint:** $\mathbf{r}_2(0) \;=\; \mathbf{r}_1(\frac{\pi}{2})$.] Next, think of the helix segment parametrized by $\mathbf{r}_2$ as being a corkscrew and the helix segment parametrized by $\mathbf{r}_1$ as being the "wormhole" the corkscrew burrows in a cork. Use the models to show how to simultaneously rotate and translate the corkscrew so that it burrows out the wormhole. Hence explain how a corkscrew works.

9. The DNA molecule is made from two right-handed helices where one helix is translated in the $z$-direction with respect to the other. Try to make such a double helix using the models.

10. **Geometric Imagination (GI) Builder:** [*Do this after class*] Work out a parametrization for the helices on which DNA is actually based. The wikipedia entry for DNA may help. In particular, what is the radius of each helix? What is the pitch? What is the $z$-translation between the two helices in DNA? *If you give me the formula and convince me it is (close to) correct, then I will make you a 3D-printed model of the double helix upon which the DNA molecule is based.*