A Menagerie of Mathematical Models

Active Learning Project #7
Max/Min/Saddle

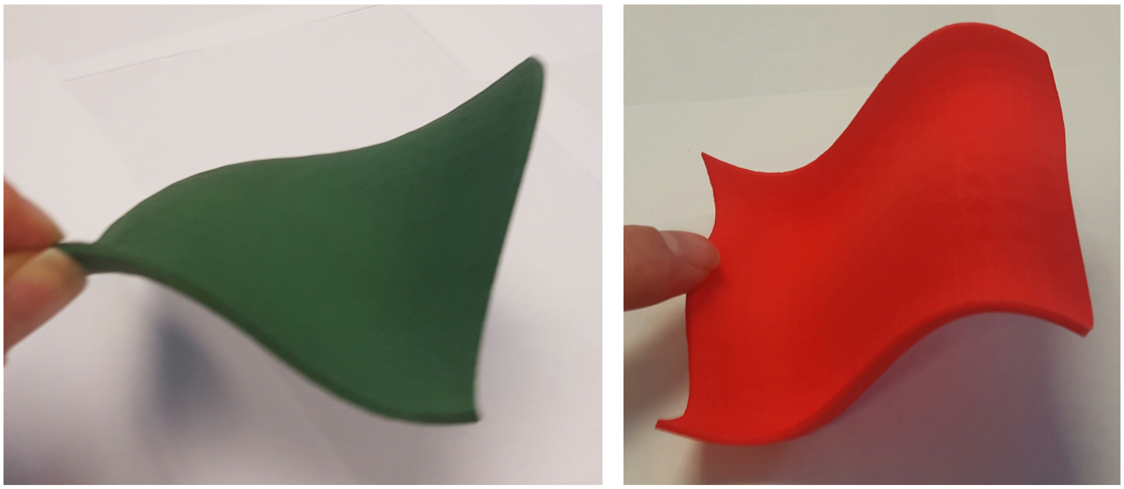

Figure 1: **Left:** Green model. This model is the graph of the function $f(x, y) = 4 + x^3 + y^3 - 3xy$. A contour map of this function is given in Stewart's Calculus Exercise 14.7.3. **Right:** Red model. This model is the graph of the function $f(x, y) = 3x - x^3 - 2y^2 + y^4$. A contour map of this function is given in Stewart's Calculus Exercise 14.7.4.

The point of this project is to identify and classify the critical points of a function $z = f(x, y)$ using a contour map and using a 3D-printed model of the surface given by the graph of the function.

Guiding principles

The following two guiding principles will help you classify the critical points of a function $z = f(x, y)$ using its contour map.

1. Near a local minimum of a function $z = f(x, y)$ the contour map looks like a rotated and warped version of the contour map of the function $z = x^2 + y^2$ near the origin.

2. Near a local maximum of a function $z = f(x, y)$ the contour map looks like a rotated and warped version of the contour map of the function $z = -x^2 - y^2$ near the origin.

3. Near a saddle point of a function $z = f(x, y)$ the contour map looks like a rotated and warped version of the contour map of the function $z = x^2 - y^2$ near the origin.

Questions

In what follows **either work with the green model or the red one.**

1. As a refresher, sketch contour maps of $z = x^2 + y^2$ and $z = x^2 - y^2$.

2. Do paper and pencil calculations to find the critical points of the function f and classify them as either local maxima, local minima, or saddle points.

3. Use the answers to your calculation to identify the critical points on the contour map given in Stewart's Calculus text.
4. Verify that the guiding principles above hold for each of the critical points you identified in the contour map. In particular for each critical point, make a sketch of the region of the contour map near critical point. Then explain how the contour map you sketched is a rotated and warped version of one of the contour maps given in the guiding principles.
5. Now identify the locations of the critical points of the function on the model. To get started you will need to correctly align the model with the horizontal xy-plane.
6. Suppose you were just given the model and knew how it was aligned with the horizontal xy-plane, but you didn't have a formula for the function. How could you identify any local maxima, local minima, and saddle points on the model?